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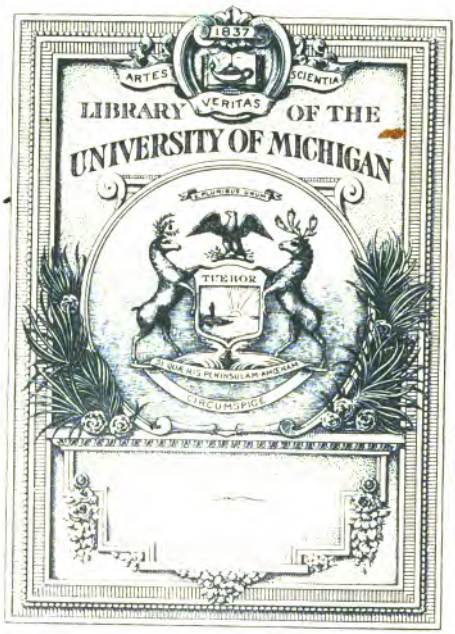
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APPENDIX

Per

HUMFREDUM DITTON.

In usum utriusque Scholæ Mathematicæ apud HOSPITIUM-CHRISTI *Londinense.*

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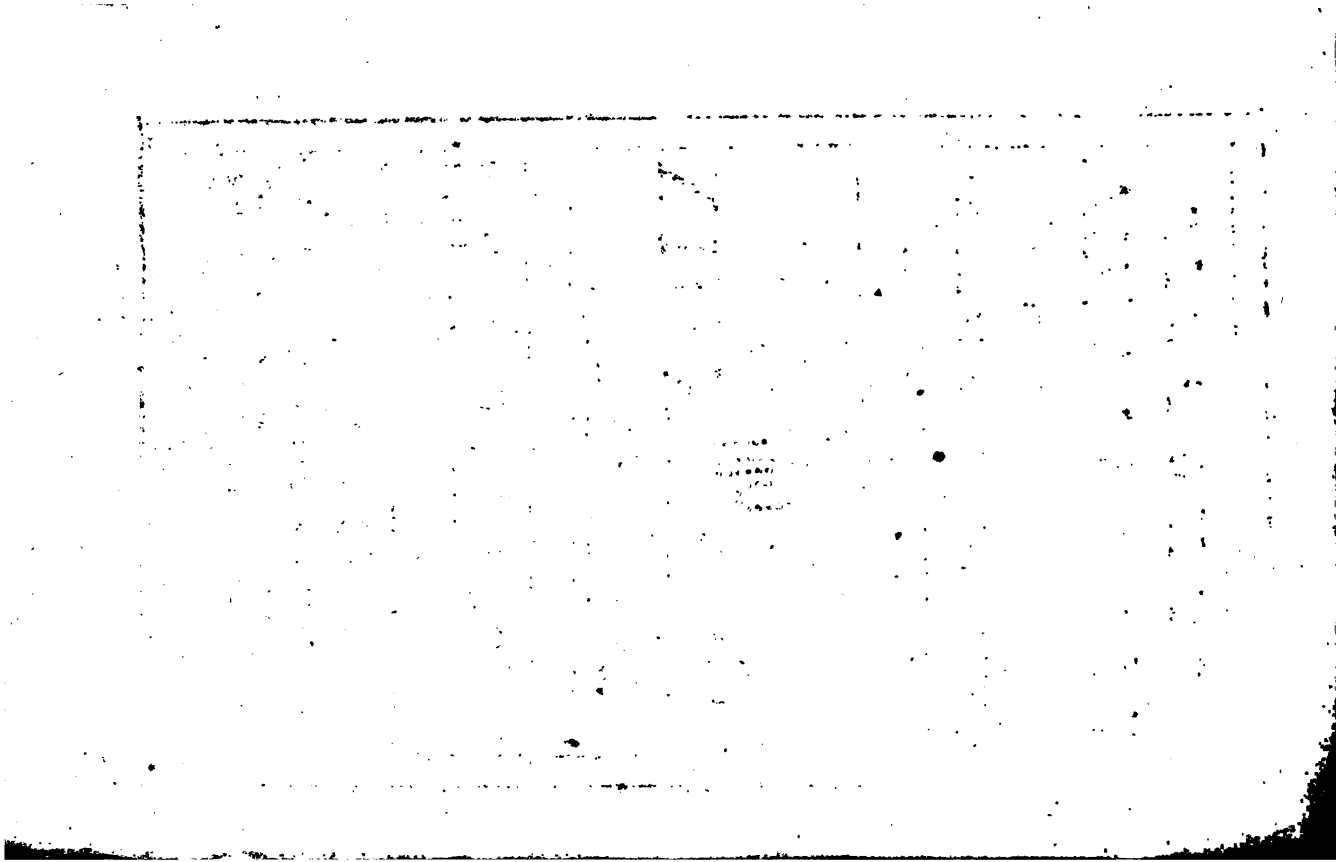

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(Cujus Sumptibus MS: comparatum & benevolentiâ dicto Hoſpitio dicatum fuit)

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In uſum utriuſque *Schola Mathematica*

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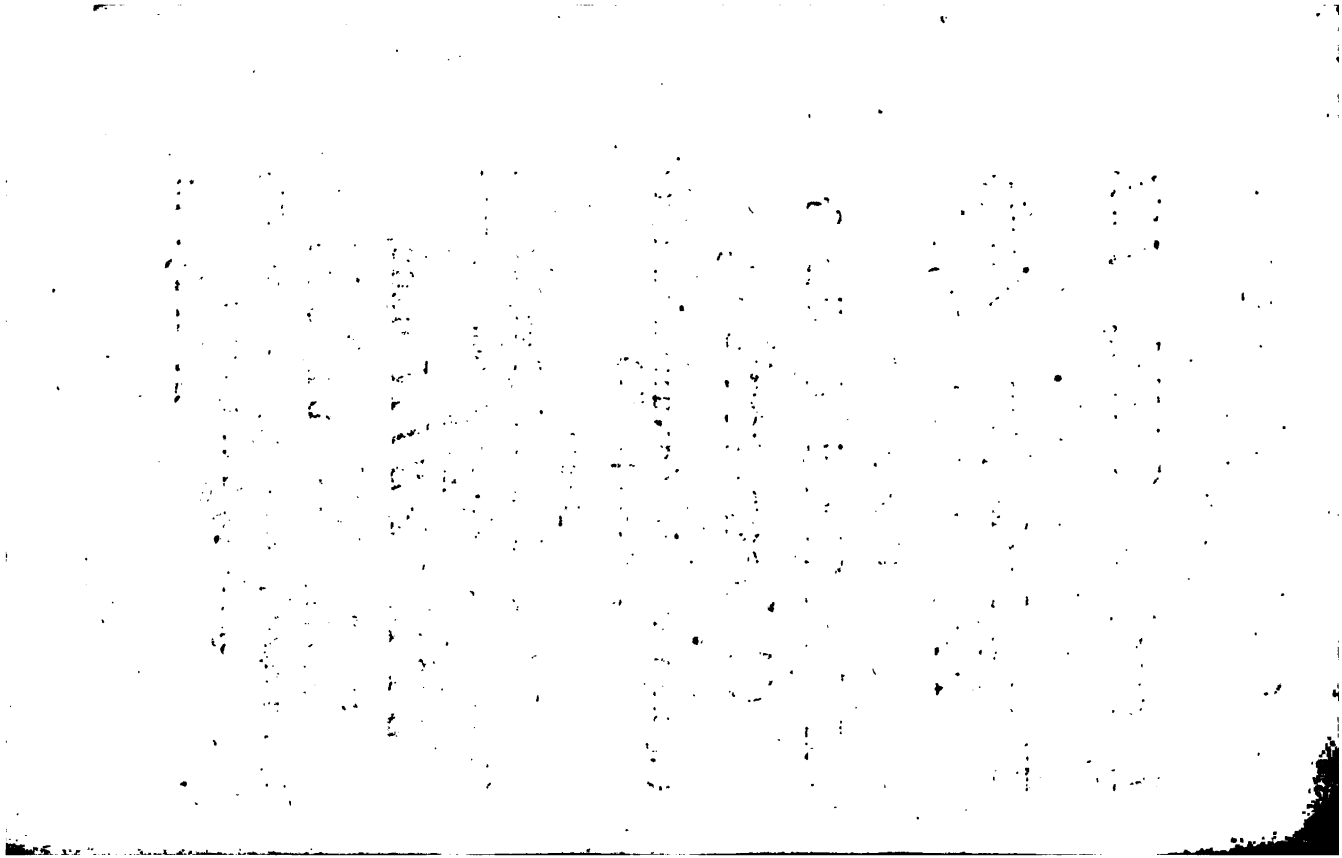

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INTRO-

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4. l. 2. pro nxm l. $n+m$. & pro $x^{b+b+c+1}$, l. x^{b+c+2}
21. l. 10. l. *esse.*
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INTRODUCTIO

AD

Algebram Specioſam.

ALgebra *Specioſa* eſt Sapientia quantitatum, ad ſolvenda Problemata & inveſtiganda Theoremata quæ in iis delitescunt. *Specioſa* dicitur ad differentiam *Numeroſæ*, & ratio appellationis paulo poſt patebit. Cæterùm alio nomine vocatur *Analytica*, necnon Matheſis univerſalis.

Quantitates [ſive ſint Numeri ſive Magnitudines] diſtinguuntur in *Datas* & *Quæſitas*.

Utræque denotari ſolent certis literis alphabeti, etſi apud diverſos Authores diverſimodè.

Nos quantitates datas deſignabimus per literas alphabeti priores, *a*, *b*, *c*, *d*, &c. quæſitas autem per poſtremas, *x*, *y*, *z*. Et quidem adhibendo literas minores, tam quia faciliùs pinguntur, tam verò ad vitandam confuſionem ; quoniam majores ſeu capitales literæ communiter adhiberi ſolent in deſignandis figurarum Geometricarum lineis.

Itaque quantitates iiſdem literis deſignatæ [verbi gratiâ *a* & *a*; *b* & *b*] in eâdem quæſtione in-

telliguntur ejuſdem eſſe valoris & ſubjecti, hoc eſt, denotant eaſdem ſeu æquales lineas vel numeros: ac proinde præfixis numeris in unum conjungi poſſunt, ut $2a$, $3b$, $4c$, $\frac{1}{2}a$, $\frac{2}{3}b$, $\frac{3}{4}c$, &c.

Quæ vero diverſis literis deſignantur, ſunt diverſi valoris; quæ non conjunguntur, niſi per modum multiplicationis, aut mediantibus ſignis $+$ vel $-$; quorum prius denotat *plus*, alterum *minus*; illud eſt nota additionis ſeu augmenti, hoc vero ſubductionis ſeu defectus.

Ac notandum, quod ſigna $+$ & $-$ ſemper quantitatibus ſuis præponuntur, adeoque ad quantitatem ſequentem referuntur; & quantitates quæ nullum ſignum præfixum habent [ut ſunt initiales] intelliguntur habere $+$.

Inter quantitates diverſi valoris poteſt tamen dari æqualitas, exempli gratia, ut duo b ſint æqualia tribus a [ſicut 5 floreni Belgici ſunt æquales 9 ſolidis Anglicanis.]

Pro denotanda autem æqualitate [quæ in Algebra utramque paginam facit] adhiberi ſolent hæc ſigna; $=$ vel $\propto$; ut $2b=3a$; hoc eſt, $2b$ æqualia $3a$, &c.

Porro Inſtrumenta, quibus Algebra ad ſolvendas quæſtiones & inquirendas veritates utitur, ſunt tum operationes Logiſticæ, tum præcipuè æquationes.

Logiſtica alia eſt Numeroſa, quæ per Numeros; alia Specioſa quæ per Species ſeu Symbola univerſalia procedit, nempe per elementa alphabeti, quibus tum numeri tum magnitudines [uti initio dictum] deſignantur.

Hinc ipſa Algebra diſtinguitur in Numeroſam & Specioſam: quarum illa vetuſtior, hæc recentior

tior eſt, & quantum conſtat, Franciſcum Vietam primum Authorem agnoſcit a cujus tempore non tantum egregie a diverſis præſtantiſſimis viris exculta, ſed jam velut ad apicem perfectionis perducta eſt.

Diſcrimen autem inter utramque Algebram potiſſimum in eo conſiſtit, quod Specioſa in quovis Problemate non duntaxat ipſum quæſitum invenit, ſicut Numeroſa; ſed inſimul generalem regulam & modum conſtruendi in lucem profert; id quod Numeroſa præſtare non valet, dum hæc plerumque operationibus ſuis veſtigia reſolutionis confundit, quæ Specioſa ſemper integra conſervat, ut eadem ad compoſitionem prompto pede repetere liceat.

Logiſtica Specioſa, perinde ut Numeroſa, conſideranda & tractanda venit in quantitatibus (1.) abſolutis iiſque integris, (2.) fractis ſeu fractionibus, (3.) ſurdis ſeu irrationalibus, (4.) binomiis.

Cæterum Logiſtica Numeroſa hoc loco partim præſupponitur, partim ex Specioſa ultro innoteſcet, cum utraque iiſdem regulis utatur.

Sequitur ergo Logiſtica quantitatum integrarum.

ADDITIO.

COnjungit ſive copulat omnes quantitates propoſitas, ſervatis iiſdem Figuris + & —.

EXEMPLA.

1. *In quantitatibus ſimplicibus ſeu ſolitariis ejuſdem nominis.*

2a	a	3d
add. a	add. —a	5d
ſumma 2a+a	ſum. a—a	ſum. 8d
hoc eſt 3a	hoc eſt o *	

* Nam quantitates ejuſdem nominis contrariis ſignis adfectæ, in add. ſe mutuo tollunt.

2c	5b	2b
add. 4c	add. —2b	add. —5b
9c	ſum. 5b—2b	ſum. 2b—5b
ſum. 15c	hoc eſt 3b	hoc eſt —3b

2. *In quantitatibus ſimplicibus diverſi nominis.*

a	4c	3c
add. b	add. —2d	add. 2e
ſum. a+b	ſum. 4c—2d	& f
		ſum. 3c+2e+f

b	3f	2d
add. —a	add. 1	add. —2
ſum. b—a	ſum. 3f+1	ſum. 2d—2

3. *In*

3. *In quantitatibus compoſitis ſeu copulatis, ejuſdem nominis.*

	$2a + 3c + 4d$		$a + b - 2c$
add.	$5d + a + 2c$	add.	$a - b + 3c$
ſum.	$3a + 5c + 9d$	ſum.	$2a + c$

Advertendum hic quantitates addendas interdum diſſimili ordine proponi, quæ tamen facile ſuo quoque loco reſtituentur. Idem quoque in Suſtractione & Diviſione uſu venire ſolet.

Ubi ſi ſigna ſint diverſa, additio fiat ſubſtrahendo, præpoſito ſigno majoris.

	$6a + 2b - d$		$4f + 2b - 5c$
add.	$-4d - 2a + 3b$	add.	$3c - f - 2b$
ſum.	$4a + 5b - 5d$	ſum.	$3f - 2c$
	$2aa + 2b - 5c + 9d$		$bb + 2abc - ab + ac$
add.	$-9d + aa - 6b + 5c$	add.	$bb - abc + ab - 3ac$
ſum.	$3aa - 4b$	ſum.	$2bb + abc - 2ac$
	$5d - 3ff + 2$		$3ff + 2d - 2$
add.	$-2d + 4ff - 6$	add.	$-4ff - 5d + 6$
ſum.	$3d + ff - 4$	ſum.	$-ff - 3d + 4$

4. *In quantitatibus compoſitis diverſi nominis.*

	$3a + c$		$5d - 4c + 1$
add.	$2b + d - f$	add.	$-a - 2b + 45$
ſum.	$3a + c + 2b + d - f$	ſum.	$5d - 4c - a - 2b + 46$

SUBTRACTIO.

CONjungit ſive copulat utraſque quantitates propoſitas, mutatis omnibus ſignis quantitatis ſubducendæ.

EXEMPLA.

1. *In quantitatibus ſimplicibus ejuſdem nominis.*

ex $5a$	ex $2bc$	ex $4acd$
ſubt. $2a$	ſubt. bc	ſubt. $4acd$
reſtant $5a-2a$	reſt. $2bc-bc$	reſt. $4acd-4acd$
hoc eſt $3a$	hoc eſt bc	hoc eſt 0
ex $-df$	ex $4b$	ex $3c$
ſubt. $-df$	ſubt. $6b$	ſubt. $-3c$
reſt. $-df+df$	reſt. $4b-6b$	reſt. $3c+3c$
hoc eſt 0	hoc eſt $-2b$	hoc eſt $6c$

2. *In quantitatibus ſimplicibus diverſi nominis.*

ex a	ex $3d$	ex c
ſubt. b	ſubt. $2c$	ſubt. $a, b, \& d$
reſt. $a-b$	reſt. $3d-2c$	reſt. $c-a-b-d$
ex $a \& b$	ex df	ex $4bc$
ſubt. $c, d, \& e$	ſubt. $-2bd$	ſubt. 4
reſt. $a+b-c-d-e$	reſt. $df+2bd$	reſt. $4bc-4$

Nota. Si dubium ſit, utra quantitatum ſit major vel minor, differentia generaliter alio quodam ſigno exhiberi ſolet, v. g.

$b \sim c$ hoc eſt $b-c$ vel $c-b$

3. *In*

3. *In quantitatibus compoſitis ejuſdem nominis.*

* Exempla ubi ſigna ſunt eadem.

ex	$4a + 3b$	ex	$6c - 5b$
ſubt.	$2a + 2b$	ſubt.	$4c - 3b$
reſt.	$2a + b$	reſt.	$2c - 2b$

ex	$3c + 2a - b$	ex	$5d + 12$
ſubt.	$c + 3a - 2b$	ſubt.	$6d - 5$
reſt.	$2c - a + b$	reſt.	$-d + 17$

ex	$ab - 4cd$	ex	$3c + 2a - d$
ſubt.	$2ab - 5cd$	ſubt.	$2c + 3a$
reſt.	$-ab + cd$	reſt.	$c - a - d$

$-2c$ ⌊ ſit $a = 9$ ∴ $4a - 2c = 32$
$-6c$ $c = 2$ & $2a - 6c = 6$
$a + 4c$ & $2a + 4c = 26$

ex $5d + 2b - c$ ⌊ ſit $d = 12$ ∴ $5d + 2b = 66$
ſub. $4d + 5b - c$ $b = 3$ $4d + 5b = 63$
reſt. $d - 3b$ $d - 3b = 3$

ex	$6a + 9b - 3c + 4d$
ſubt.	$7a + 2b - 3c + 8d$
reſtant	$-a + 7b \quad - 4d$

* Ubi ſi ſigna ſint eadem, erit & reſiduum idem; niſi cum inferior quantitas eſt major, tunc a ſubtrahendo, vice verſa, reſiduum ſortietur ſignum contrarium. Si vero ſigna ſint diverſa, ſubtractio fiet addendo, præfixo ſigno ſuperioris.

EXEMPLA ubi ſigna ſunt diverſa.

ex $4a + 3b$	ſit $a=3$	$\because 4a + 3b = 18$
ſub. $2a - 2b$	$b=2$	$2a - 2b = 2$
reſt. $2a + 5b$		$2a + 5b = 16$

ex $2b - 2c + d$	ex $5bc - bb$
ſub. $b + 4c - d$	ſub. $3bb - 3bc$
reſt. $b - 6c + 2d$	reſt. $8bc - 4bb$

ex $aa - 4d + 6c - 2a$
ſub. $4a + 2aa - 5c - 6d$
reſt. $2d - aa + 11c - 6a$

ex $6bb + 3aa - 3c + 9b$
ſubt. $4bb - 4aa - 6c - 5b$
reſt. $2bb + 7aa + 3c + 14b$

ex $ccc + bbd - aac - eee$
ſubt. $2bbd - 2ccc + eee$
reſt. $3ccc - bbd - aac - 2eee$

ex $7ab + dd - 10$
ſubt. $9ab - 2dd + 18$
reſt. $-2ab + 3dd - 28$

4. *In*

4. *In quantitatibus compoſitis diverſi nominis.*

ex $aa-bb$	ex $bc-dd$
ſubt. $aa-ab+ac$	ſubt. $ad+bd-2cd$
reſt. $ab-bb-ac$	reſt. $bc-dd-ad.bd+2cd$

ex $5b-2a$ ſubtr. $3c+d-4f$
reſt. $5b-2a-3c-d+4f$

ſit $b=10$, $a=5$, $c=6$, $d=7$, $f=4$ ∴

$5b-2a=40$
$3c+d-4f=9$
$5b-2a-d+4f-3c=31$
nam $5b+4f=66$
ſubt. $2a+3c+d=35$
reſt. 31

MULTI-

MULTIPLICATIO.

COnnectit [immediate] literas utriuſque factoris; ſeu apponit multiplicantem ipſi multiplicando. At numeri quantitatibus præfixi multiplicantur more vulgari. Porro ſigna eadem [ſive + fuerit ſive −] dant in productò +, ſed ſigna diverſa dant −.

EXEMPLA.

1. *In quantitatibus ſimplicibus.*

Multipl. a
in b
product. erit ab

Multipl. a
in a
product. aa vel a^2
hoc eſt a quadratum
ſeu a ſecundanum

mult. aa ſeu a^2
in a
prod. aaa ſeu a^3
hoc eſt a cubicum
ſeu a tertianum

mult.

mult. d in b in a
db
a
dba productum

mult. c in 12
prod. $12c$

mult. $4a$
in $2b$
prod. $8ab$

mult. $3bd$
in $2ab$
$6bbad$

mult. c^3 in $5d^2$
prod. $5d^2c^3$

mult. $5cd$ in $3ab$
prod. $15abcd$

mult. $2ac$ in 10
prod. $20ac$

mult. $6b^4$ in $3b^2$
prod. $18b^6$

mult. $4ab^2$ in $4ca^2$
prod. $16a^3b^2c$

mult. $9aacdf$
in $5accf$
prod. $45a^3c^3f^2d$

Ubi accedit productorum partialium additio ſeu collectio [more vulgari] ut habeatur productum totale.

mult. $a+b$ in c
prod. $ac+bc$

mult. $b-a$ in $2d$
prod. $2bd-2ad$

mult. $a+2b-3c+d$ in $3e$
prod. $3ae+6be-9ce+3de$
mult. $6b-2a-c-4f$ in $2ab$
prod. $12ab^2-4a^2b-2abc-8abf$
mult. $c+b$ in $a+d$
prod. $ac+ab+cd+bd$

mult.

SUBTRACTIO.

CONjungit ſive copulat utraſque quantitates propoſitas, mutatis omnibus ſignis quantitatis ſubducendæ.

EXEMPLA.

1. *In quantitatibus ſimplicibus ejuſdem nominis.*

ex $5a$	ex $2bc$	ex $4acd$
ſubt. $2a$	ſubt. bc	ſubt. $4acd$
reſtant $5a-2a$	reſt. $2bc-bc$	reſt. $4acd-4acd$
hoc eſt $3a$	hoc eſt bc	hoc eſt 0
ex $-df$	ex $4b$	ex $3c$
ſubt. $-df$	ſubt. $6b$	ſubt. $-3c$
reſt. $-df+df$	reſt. $4b-6b$	reſt. $3c+3c$
hoc eſt 0	hoc eſt $-2b$	hoc eſt $6c$

2. *In quantitatibus ſimplicibus diverſi nominis.*

ex a	ex $3d$	ex c
ſubt. b	ſubt. $2c$	ſubt. a, b, & d
reſt. $a-b$	reſt. $3d-2c$	reſt. $c-a-b-d$
ex a & b	ex df	ex $4bc$
ſubt. c, d, & e	ſubt. $-2bd$	ſubt. 4
reſt. $a+b-c-d-e$	reſt. $df+2bd$	reſt. $4bc-4$

Nota. Si dubium ſit, utra quantitatum ſit major vel minor, differentia generaliter alio quodam ſigno exhiberi ſolet, v. g.

$b \sim c$ hoc eſt $b-c$ vel $c-b$

3. *In*

3. *In quantitatibus compoſitis ejuſdem nominis.*

* Exempla ubi ſigna ſunt eadem.

ex	$4a + 3b$	ex	$6c - 5b$
ſubt.	$2a + 2b$	ſubt.	$4c - 3b$
reſt.	$2a + b$	reſt.	$2c - 2b$

ex	$3c + 2a - b$	ex	$5d + 12$
ſubt.	$c + 3a - 2b$	ſubt.	$6d - 5$
reſt.	$2c - a + b$	reſt.	$-d + 17$

ex	$ab - 4cd$	ex	$3c + 2a - d$
ſubt.	$2ab - 5cd$	ſubt.	$2c + 3a$
reſt.	$-ab + cd$	reſt.	$c - a - d$

ex	$4a - 2c$	ſit $a = 9$	$\because 4a - 2c = 32$
ſub.	$2a - 6c$	$c = 2$	$\&\ 2a - 6c = 6$
reſt.	$2a + 4c$		$\&\ 2a + 4c = 26$

ex	$5d + 2b - c$	ſit $d = 12$	$\because 5d + 2b = 66$
ſub.	$4d + 5b - c$	$b = 3$	$4d + 5b = 63$
reſt.	$d - 3b$		$d - 3b = 3$

ex	$6a + 9b - 3c + 4d$
ſubt.	$7a + 2b - 3c + 8d$
reſtant	$-a + 7b \quad - 4d$

* Ubi ſi ſigna ſint eadem, erit & reſiduum idem; niſi cum inferior quantitas eſt major, tunc a ſubtrahendo, vice verſa, reſiduum ſortietur ſignum contrarium. Si vero ſigna ſint diverſa, ſubtractio fiet addendo, præfixo ſigno ſuperioris.

EXEMPLA ubi signa sunt diversa.

ex $4a+3b$	∴ sit $a=3$	∵ $4a+3b=18$
sub. $2a-2b$	$b=2$	$2a-2b=2$
rest. $2a+5b$		$2a+5b=16$

ex $2b-2c+d$	ex $5bc-bb$
sub. $b+4c-d$	sub. $3bb-3bc$
rest.. $b-6c+2d$	rest. $8bc-4bb$

ex $aa-4d+6c-2a$
sub. $4d+2aa-5c-6d$
rest. $2d-aa+11c-6a$

ex $6bb+3aa-3c+9b$
subt. $4bb-4aa-6c-5b$
rest. $2bb+7aa+3c+14b$

ex $ccc+bbd-aac-eee$
subt. $2bbd-2ccc+eee$
rest. $3ccc-bbd-aac-2eee$

ex $7ab+dd-10$
subt. $9ab-2dd+18$
rest. $-2ab+3dd-28$

4. *In*

4. *In quantitatibus compoſitis diverſi nominis.*

ex $aa-bb$	ex $bc-dd$
ſubt. $aa-ab+ac$	ſubt. $ad+bd-2cd$
reſt. $ab-bb-ac$	reſt. $bc-dd-ad.bd+2cd$

ex $5b-2a$ ſubtr. $3c+d-4f$
reſt. $5b-2a-3c-d+4f$

ſit $b=10$, $a=5$, $c=6$, $d=7$, $f=4$ ∴ $5b-2a=40$, $3c+d-4f=9$

$5b-2a-d+4f-3c=31$

nam $5b+4f=66$

ſubt. $2a+3c+d=35$

reſt. 31

MULTI-

MULTIPLICATIO.

COnnectit [immediate] literas utriuſque factoris; ſeu apponit multiplicantem ipſi multiplicando. At numeri quantitatibus præfixi multiplicantur more vulgari. Porro ſigna eadem [ſive + fuerit ſive −] dant in producto +, ſed ſigna diverſa dant −.

EXEMPLA.

1. *In quantitatibus ſimplicibus.*

Multipl. a
in b
product. erit ab

Multipl. a
in a
product. aa vel a^2
hoc eſt a quadratum
ſeu a ſecundanum

mult. aa ſeu a^2
in a
prod. aaa ſeu a^3
hoc eſt a cubicum
ſeu a tertianum

mult.

mult. d in b in a
db
a
dba productum

mult. c in 12
prod. $12c$

mult. $4a$
in $2b$
prod. $8ab$

mult. $3bd$
in $2ab$
$6bbad$

mult. c^3 in $5d^2$
prod. $5d^2c^3$

mult. $5cd$ in $3ab$
prod. $15abcd$

mult. $2ac$ in 10
prod. $20ac$

mult. $6b^4$ in $3b^2$
prod. $18b^6$

mult. $4ab^2$ in $4ca^2$
prod. $16a^3b^2c$

mult. $9aacdf$
in $5accf$
prod. $45a^3c^3f^2d$

Ubi accedit productorum partialium additio seu collectio [more vulgari] ut habeatur productum totale.

mult. $a+b$ in c
prod. $ac+bc$

mult. $b-a$ in $2d$
prod. $2bd-2ad$

mult. $a+2b-3c+d$ in $3e$
prod. $3ae+6be-9ce+3de$
mult. $6b-2a-c-4f$ in $2ab$
prod. $12ab^2-4a^2b-2abc-8abf$
mult. $c+b$ in $a+d$
prod. $ac+ab+cd+bd$

mult.

mult. $b - a$ in $c - d$
prod. $bc - ac - bd + ad$
mult. $a + b$ in $a + b$
prod. $a^2 + 2ab + b^2$
mult. $a - b$ in $a - b$
prod. $a^2 - 2ab + b^2$
mult. $b + c$ in $b - c$
prod. $b^2 - c^2$
mult. $b + c - 2d$ in $b + c - 2d$
prod. $b^2 + 2bc - 4bd + c^2 - 4cd + 4d^2$
mult. $2a + 4b$ in $3c - 5d$
prod. $6ac + 12bc - 10ad - 20bd$
mult. $6f - d - 2$ in $f + 2d$
prod. $6f^2 + 11df - 2d^2 - 2f - 4d$
mult. $3b^2 - 2a^2 + 5$ in $9a + 4b - 1$
prod. $27ab^2 - 18a^3 + 45a + 12b^2$, &c.

Cæterum notandum multiplicationem interdum ſymbolice tantum indigitari hoc modo.

$$\begin{array}{rl} & a + b - c \times d - e \\ \text{vel} & a + b - c \times d + e \\ \text{vel ita} & a + b - c : d + e \end{array}$$

DIVI-

DIVISIO.

REſolvit dividendum in diviſorem & quotum: quo in totum diviſorem ducto reſtituatur exhauriaturque ipſe dividendus. Cæterum cum numeris prefixis diviſio inſtituitur more vulgari, & ſigna eadem dant in quoto -|-, diverſa dant —, ut in multiplicando.

Vel Diviſio ſubducit quaſi ſeu extrahit diviſorem ex dividendo, & quod relinquitur aſſumit pro quoto.

Et hoc quidem ſufficit cum diviſor eſt ſimplex; ſed ſi is fuerit compoſitus, inquiſitio quoti tantum inſtituitur cum prima nota diviſoris, perinde ut in praxi numercia fieri ſolet.

EXEMPLA.

1. *In quantitatibus ſimplicibus.*

divid. aa per a | a quotus. divid. bc per b | c quotus.

divid. $2abc$ per $2b$ | ac quotus.

divid. $9cd$ per $3c$ | quotus $3d$

divid. $3f$ per f | quotus 3

divid. $4a^2b^2$ per $2b^2$ | quotus $2a^2$

divid. $6c^3b^4$ per $2c^2b^3$ | quotus $3bc$

divid. $3a^2b^3$ per $3ab^2$ | quotus ab

divid. $18cd$ per 9 | quotus $2cd$

divid. $4a^3b^2c$ per $5a^2b$ | quotus $\frac{4}{5}$ abc ſeu $\frac{4abc}{5}$

divid. $6b^3c^3d^4$ per $4bc^2d^3$ | quotus $1\frac{1}{2}bcd$

divid. $12a^3c^4d^5$ per $4a^2c$ | quotus $3a^2c^3d^5$

divid. $5df$ per $5df$ | quotus erit 1. Nam ſi diviſor ſit=dividendo, quotus ſemper erit unitas.

2. *In*

2. *In quantitatibus compoſitis.*

divid. $ab + bc$ per b | quotus erit $a + c$
divid. $bc - cd$ per c | quotus $b - d$
divid. $af + ag + df + dg$ per $f + g$ | quot. $a + d$
divid. $bd - bc - ad + ac$ per $d - c$ | quotus $b - a$
div. $6a^2b + 12abc - 10a^2c^2 - 20ac^3$ per $2a^2 + 4ac$ | qu. $3b - 5c^2$
divid. $d^2 + d$ per $d + 1$ | quotus d
divid. $2f^3 - f^2$ per $2f - 1$ | quotus f^2
divid. $a^2 - b^2$ per $a + b$ | quotus $a - b$
divid. $a^2 - b^2$ per $a - b$ | quotus $a + b$
divid. $b^3 - 3b^2a + 3ba^2 - a^3$ | quotus $b^2 - 2ab + a^2$
per $b - a$
ſubtr. $b^3 - b^2a$
reman. $-2b^2a + 3ba^2 - a^3$ | $-2ba$
$b - a$
ſubtr. $-2b^2a + 2ba^2$
reman. $+ba^2 - a^3$ | $+a^2$
$b - a$
ſubtr. $+ba^2 - a^3$
div. $8ab^3 - 12b^2cd + 16b^6 + 10abc^3 - 15c^4d + 20b^4c^3$
per $4b^2 + 5c^3$
1. membrum] $8ab^3 + 10abc^3$ | $2ab$
2. membrum] $-12b^2cd - 15c^4d$ | $-3cd$ } quotus
3. membrum] $+16b^6 + 20b^4c^3$ | $+4b^4$ }

[Examen per multiplicationem.]

divid. $c^3 + d^3$ per $c + d$ | quotus $c^2 + d^2 - cd$
divid. $ab + 3b - 2a - 6$ per $b - 2$ | quotus $a + 3$

Porro

Porro ſi diviſio modo præcedenti fieri nequeat ſubſcripto diviſore ipſi dividendo conſtituetur fractio, ut patet ex ſequentibus,

divid. $\left\{\begin{matrix} ac \text{ per } b \\ ef \text{ per } bc \\ c \text{ per } d \\ def \text{ per } gh \end{matrix}\right\}$ quotus erit $\left\{\begin{matrix} \frac{ac}{b} \\ \frac{af}{bc} \\ \frac{c}{d} \\ \frac{def}{gh} \end{matrix}\right.$

Sic $a+b$ per d, erit $\frac{a+b}{d}$

b^2+c^2 per $e+f$ $\frac{b^2+c^2}{e+f}$

$2a+3c-d$ per $b+c+d$ $\frac{2a+3c-d}{b+c+d}$

$dbc+af$ per bc | $d+\frac{af}{bc}$

$bde+3c$ per d | $be+\frac{3c}{d}$

$efg+fg$ per ef | $g+\frac{fg}{ef}$ hoc eſt $\frac{g}{e}$

§ Pari modo diviſio quævis ſymbolice indigitari ſolet, etiamſi illa reipſa fieri poſſit, ut

$$\frac{a^2-b^2}{a+b}$$

REGULA

Regula PROPORTIONUM

EX datis duobus vel tribus terminis rite diſpoſitis, ope multiplicationis & diviſionis, perinde ut in praxi numeroſa, eruit tertium vel quartum proportionalem quæſitum.

Et quantitatum in ſerie proportionali diſpoſitio denotari ſolet hoc modo;

(1)	(2)	(2)	(3)
a.	*b*::	*b.*	X

hoc eſt, ut *a* ad *b*: ita *b* ad X.

(1)	(2)	(3)	(4)
a.	*b*::	*c.*	X

ut *a* ad *b*: ita *c* ad X.

Vel etiam ſic;

a ∴ *b*: *b* ∴ X
a ∴ *b*: *c* ∴ X

Porro quoad ipſam praxin notandum

1.) Si in termino primo & ſecundo; vel in primo & tertio, eædem literæ occurrant, illas per modum diviſionis ante operationem expungi; quod ipſum etiam de præfixis numeris intelligendum.

2.) Si vero primus terminus ſit diverſus à reliquis [ſaltem ex parte] uti plerumque fit, quartum quæſitum ſemper in forma fractionis prodire.

EXEMPLA.

Exempla in Quantitatibus ſimplicibus.

Termini dati.

1.) $a.$ $2a :: 2a$

$4aa$ | $4a$ tertius quæſitus.
x quo præſuppoſ.$=x$
erit $x = 4a$

Vel compendioſus ſic:

x $2x :: 2a$

$4aa$ tertius quæſitus.

2.) $b.$ $ab :: ab$

$\frac{aabb}{b}$ | aab tertius quæſitus.

vel ita, b $ab :: ab$ | aab

3.) $2c.$ $d :: d$ | $\frac{dd}{2c}$ tertius quæſitus.

4.) $3a.$ $b :: 3b$ | $\frac{bb}{a}$ tertius quæſitus.

5.) $x.$ $c :: 2xb$

$2bc$ quartus quæſitus.

6.) $2b.$ $3a :: \frac{4bd}{x}$

$6ad$ quartus quæſitus.

7.) $d.$ $5b :: a$ | $\frac{5ab}{d}$ quartus quæſitus.

8.) $\cancel{1}\cancel{0}a \quad 7c :: \cancel{1}\cancel{0}d^2$ | $\frac{7cd^2}{a}$ quart. quæsit.

Exempla in Quantitatibus compositis.

9.) $a. \quad a+b :: a+b$

$aa+2ab+bb$ | $a+2b+\frac{bb}{a}$ tertius quæsitus.

div. a

10.) $a. \quad a+b :: a-b$

$aa-bb$ | $a-\frac{bb}{a}$ quartus quæsitus.

div. a

11.) $b. \quad b+c :: 2b-c$

$2bb+bc-cc$ | $2b+c-\frac{cc}{b}$ quartus quæsitus.

div. b

12.) $a+2b \quad a :: 2a-b$

| $\frac{2aa-ab}{a+2b}$ quart. quæsit.

13.) $\cancel{3}b-\cancel{3}a \quad 2a+c :: \cancel{3}d$

$\frac{2ad+cd}{b-a}$ quartus quæsitus.

14.) $2d+c \quad 2b+d :: 2d-c$

$\frac{4bd+2dd-2bc-cd}{2d+c}$ quart. quæsit.

GENE-

GENESIS & ANALYSIS POTESTATUM.

[*Ab aliis Involutio & Evolutio dictæ, adhibitis hisce notis* ⓢ *ɯ*]

POteſtates nihil aliud ſunt quam producta ex continuâ alicujus aſſumpti lateris ſeu radicis multiplicatione orta.

Nam ſuppoſito, quod quælibet radix ſit ſui ipſius prima poteſtas; vel, quod prima poteſtas oriatur, cum radix ducitur in unitatem;

Si deinde eadem radix ducatur

in { ſeipſam / ſuum Quadratum / ſuum Cubum / ſuum Quadrato-quadratum / ſuum Quadrato cubum }

producitur eusdem { Quadratum / Cubus / Quadrato-quadr. / Quadrato-cubus / Cubo-cubus } quæ eſt poteſtas { Secundana / Tertiana / Quartana / Quintana / Sextana }

quarum proinde notæ ſeu indices numerales ſunt { 2 / 3 / 4 / 5 / 6 } vel quælibet aliæ ex his ortæ literales vero hæ { $\frac{Q}{C}$ / $\frac{Qq}{Qc}$ / $\frac{Cc}{Cc}$ }

Et ſic porro in infinitum, pro numero ſcilicet dimenſionum ſuarum, ex quibus componuntur.

Quo autem magis compoſitæ ſunt poteſtates, eò cenſentur altiores; ac proinde non tantum denominatione ſed ſimul gradibus ſuis invicem diſtinguuntur: qui ipſi denique ſcalam aliquam progreſſionalem conſtituunt, in qua quantitates velut aſcendant ac deſcendant.

Sed & notandum, quod Poteſtates quæ ſunt ſupra Cubicam, non tantum ab ipsâ radice, in gradum proxime præcedentem ductâ, ſed etiam ex poteſtatum inferiorum mutuâ multiplicatione procreentur, quod & ipſa nomina indicant. Exempli gratiâ, Quadrato quadratum oritur non tantum ex ducta radicis in cubum, ſed etiam ex multiplicatione quadrati in ſeipſum; quò fit ut hæc poteſtas [quartana] etiam ſit quadratum. Idem eſto judicium de cæteris.

Porro omnis radix vel eſt (1) ſingularis, hoc eſt unius nominis, ut a, vel $2a$, vel $3a$, &c. nam numeri literis præfixi non augent nomen, vel (2) compoſita ex pluribus nominibus hoc eſt binomia, trinomia, &c. ut $a+b$, vel $a-b$, $a+b+c$, $a+b-c$, $a-c+b$, $a-b-c$, &c.

Deinde omnis radix vel eſt rationalis, ut a, $2a$, $\frac{1}{2}a$, &c. vel irrationalis, ut $\sqrt{a}$, $\sqrt{ab}$, & quales ſunt radices, tales etiam ſunt earum poteſtates; hoc autem loco tantum de rationalibus agitur.

Geneſis alicujus poteſtatis eſt ejuſdem ex datâ radicê per multiplicationem procreatio.

Analyſis alicujus poteſtatis eſt radicis ex datâ poteſtate [ope diviſionis peculiaris] extractio.

Itaque in *Geneſi* datur radix, & quæritur poteſtas. In *Analyſi* contra datur poteſtas & quæritur radix.

Tum

Tum vero radix alicujus poteſtatis [utut rationalis] non ſemper eſt latus proprie dictum ſeu linea [vel poteſtas prima] ſed ſæpe ipſæ poteſtates [ortæ] loco & nomine radicis veniunt. Sic aa, ſeu a^2 [poteſtas ſecundana] poteſt inſervire pro radice cujuſcumque poteſtatis conſtituendæ, &c.

Itaque GENESIS poteſtatum à radice ſingulari facilis eſt: Nimirum (1) ſi radix ſit latus [ſeu poteſtas prima] apponatur eidem [ſuperius verſus dextram] nota numerica ſimplex, optatæ poteſtati competens, tanquam multiplicationis requiſitæ index: Ut ſi radix ſit a, ejus quadratum erit a^2; cubus a^3, &c. Si radix ſit $2a$, quadratum erit $4a^2$, cubus $8a^3$, &c. Numeri enim literis præfixi multiplicantur ſeorſim, ut ſuo loco dictum.

Si vero (2) radix ſit poteſtas [orta] tum index ejuſdem multiplicetur per indicem [ſimplicem] poteſtatis conſtituendæ, hoc eſt, pro conſtituendo quadrato duplicetur, pro cubo triplicetur, &c. ut ſi radix ſit a^2, ejus quadratum erit a^4, cubus a^6, quadrato quadratum a^8, &c.

Intelligitur hinc, quod indices poteſtatum alii ſint ſimplices ſeu primi, ut 2, 3, 4, &c. alii ex prioribus per multiplicationem compoſiti, primis ſuis æquivalentes; ſic index quadrati ſeu poteſtatis ſecundanæ non tantum eſt 2, ſed quilibet numerus ex binario compoſitus, ut 4, 6, 8, &c. Quo fit ut eadem quantitas diverſos poteſtatum gradus ſubire poſſit; ſic a^6 non tantum eſt poteſtas ſextana, ſed etiam tertiana, & ſecundana, &c.

Quod ſi denique (3.) ex duabus vel pluribus poteſtatibus ab eadem radice ortis, alia poteſtas componenda ſit, fiet hoc, indices datarum poteſtatum addendo, & aggregatum ipſi radici ſuperimponendo. Dentur exempl. gr. poteſtates a^7 & a^5; poteſtas ex his conflata erit a^{12}. Sic ex a^3, a^4, & a^5 prodibit a^{12}.

Neque difficilis eſt, etſi operoſior, poteſtatum procreatio à radice compoſitâ, puta binomiâ, &c.

Sit ex. gr. radix $a+b$

$a+b$ multipl.

a^2+ab
$+ab+b^2$

erit $a^2+2ab+b^2$ Quadratum

$a+b$ mult.

$a^3+3a^2b+ab^2$
$+a^2b+2ab^2+b^3$

$a^3+2a^2b+3ab^2+b^3$ Cubus

$a+b$

$a^4+3a^3b+3a^2b^2+ab^3$
$+a^3b+3a^2b^2+3ab^3+b^4$

$a^4+4a^3b+6a^2b^2+4ab^3+b^4$ Quadrato quadratum.

Atque hoc modo à radicibus compoſitis diverſæ poteſtatum tabulæ conſtrui poſſunt, quouſque libuerit.

Cæterum

Cæterum poteſtates à radice quâcunque compoſitâ productæ per ſingulos gradus habent ſuas certas proprietates, tum quoad ipſas partes, ex quibus componuntur, earamque genituras, tum quoad ſigna + & —. De quibus & aliis huc ſpectantibus videatur Clavis *Oughtredi*.

Sic fuit poteſtatum *Geneſis*; quâ probè intellectâ in praxi ſpeciosâ [adhibitis, in quantum fieri poteſt, figuris] proclivis etiam via erit ad ANALASIN, quæ nihil aliud eſt quam Inventio ſeu Extractio radicis ex quâlibet poteſtate propoſitâ; quin eadem ad praxin numeroſam loco regulæ & declarationis, magno ſane verborum compendio, inſerviet.

EXTRACTIO
Radacis Quadratæ.

[*Cujus nota eſt √q. vel etiam √ abſolute.*]

(1.) *Ex quantitatibus ſimplicibus.*

Quæ fit dividendo indicem poteſtatis propoſitæ per 2, & ſi quadratum numericum prefixum ſit, ex eo ſeorſim radicem extrahendo.

Ex $\begin{cases} a^2 \\ b^2 \\ c^2 \end{cases}$ rad. quad. eſt $\begin{cases} a \\ b \\ c \end{cases}$ Ex $\begin{cases} 4a^2 \\ 9b^2 \\ 16d^2 \end{cases}$ $\sqrt{q}$. eſt $\begin{matrix} 2a \\ 3b \\ 4d \end{matrix}$

Sic ex $\begin{cases} a^2b^2 \\ c^4 \\ 25d^6 \end{cases}$ rad. quadr. eſt $\begin{matrix} ab \\ c^2 \\ 5d^3 \end{matrix}$

(2.) *Ex Quantitatibus Compoſitis.*

Ubi Praxis ita ſe habet:

Extr. $\sqrt{q}$. ex $a^2+2ac+c^2$ | $a+c$ rad. quæſita.
a^2+2a+c
c mult.
$2ac+c^2$ ſubtr.

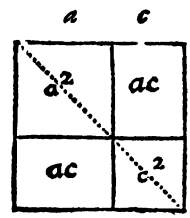
1. Po-

1.) Ponatur radix Quadrati diagonalis a^2, quæ eſt a, & ipſum quadratum a^2 deleatur per modum Subtractionis.

2.) Duplicetur inventa radicis nota a, duplum erit $2a$, quod ſcribatur ſub $2ac$ [aggregato complementorum] fiat diviſio [$2ac$ per $2a$] quotus c, erit altera nota tum radicis, tum etiam diviſoris, hactenus imperfecti, ut nimirum totus diviſor ſit $2a+c$, quî demum multiplicetur in quotum ſuum c, & productum ſubtrahatur ex ſupraſcripto; (3.) ubi cum tandem nihil relinquitur.

[*Nota:* Duplicatio radicis vocari ſolet genitura, puta diviſoris requiſiti; ſicut in extr. √ cub. triplicatio quadrati, &c.]

3.) Quod ſi porro plures adhuc notæ radicis eruendæ reſtent [ut Exemplo ultimo] duplicatio notarum præcedentium inventarum dictaque ſimul diviſio continuetur ad finem uſque; ubi cum tandem nihil relinquitur, indicium eſt Quadrati rationalis & radicis rite inventæ.



Ratio operationis facile patet ex ipſa Geneſi & appoſitâ Figurâ. Nam primam notam radicis dat Quadratum diagonale a^2; quo ablato reſtat gnomon, conſtans ex duplici complemento, ac & altero diagnoli c^2. Cum igitur utrumque complementum contineatur ſub utrâque parte radicis [hoc eſt ſub a & c, ſequitur

tur ſi aggregatum complementorum [$2ac$] dividatur per primam notam radicis duplicatam ut [$2a$] quod tunc in Quoto emerget altera radicis nota [c]. Quæ etſi in praxi ſpeciosâ ex ipſo ſtatim altero diagonali innoteſcat, tamen in praxi numerosâ ope dicti diviſoris [$2a$] inquirenda eſt & acquirenda; utut diviſor hic ſit imperfectus uſque dum ipſe Quotus eidem jungatur.

§. Ex $a^2 - 2ab + b^2$ | $a - b$ √ quæſita.

$a^2 + 2a - b$

$- b$ mult.

$- 2ab + b^2$ ſubtr.

§. Ex $4a^2 + 8ab + 4b^2$ | $2a + 2b$ √ quæſ.

$4c^2 - 16cd + 16d^2$ | $2c - 4d$ √ quæſ.

§. Ex $b^2 + 2bc + c^2 + 2bd + 2cd + d^2$ | $b + c + d$ √ quæſita.

$b^2 + 2b + c$ mult.

c

$+ 2bc + c^2$ ſubtr.

$+ 2b + 2c + d$ mult.

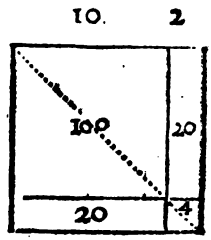
$+ d$

$+ 2bd + 2cd + d^2$ ſubtr.

Applicatio

Applicatio dictorum in Numeris.

Sit extrahenda √*q.* ex 144 quæ quidem nota est nimirum 12, hæc ita divisa intelligatur, ut prima nota (1.) pro valore loci sui [10] constituat unam partem, altera vero [2.] reliquam, & tunc procedatur ut supra.



144	10+2 hoc est 12 √ quæsita.
100	10
44	2
20	12
2	
44	

Cæterum idem obtinebitur [etsi non pari commoditate] radice 12 divisa utcunque, puta in 9 & 3, in 8 & 4, &c.

EXTRA-

EXTRACTIO
Radicis Cubicæ.

[*Cujus nota eſt hæc,* √ cub. *vel* √ c.]

(1.) *Ex Quantitatibus ſimplicibus.*

Quæ fit dividendo indicem poteſtatis propoſitæ per 3, & ſi Cubus numericus præfixus ſit, ex eo ſeorſim radicem cubicam extrahendo.

$$\text{Ex} \begin{cases} a^3 \\ b^3 \\ 8c^3 \\ 27d^3 \end{cases} \text{ rad. cub. eſt } \begin{cases} a \\ b \\ 2c \\ 3d \end{cases}$$

$$\text{Ex} \begin{cases} a^3d^3 \\ 125b^3c^3 \\ 64b^6 \end{cases} \text{ rad. cub. eſt } \begin{cases} ad \\ 5bc \\ 4b^2 \end{cases}$$

(2.) *Ex Quantitatibus compoſitis.*

Ubi Praxis ita ſe habet:

Extr. $\sqrt{c}$. ex $b^3+3b^2c+3bc^2+c^3 \mid b+c$ $\sqrt{c}$. quæſ.

mult. $\dfrac{b^3+3b^2+3b \mid +c^3}{(+c \quad +c^2}$

$+3b^2c+3bc^2+c^3$

1. Ponatur

1.) Ponatur radix Cubica Cubi diagonalis b^3, quæ eſt b, & ipſe Cubus b^3 per ſubtractionem deleatur.

2.) Inventa radicis nota b triplicetur, & triplum $3b$ ſcribatur ſub $3bc^2$, tanquam termino ſeu complemento correſpondente. Tum idem triplum $3b$ multiplicetur per dictam radicis notam b, productum erit $3b^2$, quod ſcribatur ſub complemento correſpondente $3b^2$; ſic habebitur pars diviſoris. Dividatur itaque $3b^2c$ per $3b^2$, quotus c erit altera nota radicis: Quâ multiplicatâ tum in triplum præcedentis notæ b, ut fiat $3bc$, tum in ſeipſam ut fiat Quadratum c^2, emerget totus diviſor $3b^2+3bc+c^2$, qui demum multiplicetur in quotum ſuum c, & productum ſubtrahatur ex ſupraſcripto. [Vel quod idem erit, inventa radicis nota c, ducatur in $3b^2$, & ejus Quadratus c^2 in $3b$, quibus denique jungatur ejuſdem Cubus c^3.]

3.) Si plures notæ radicis eruendæ reſtent, triplicentur notæ inventæ omnes, & triplum ſubſcribatur locis convenientibus, tum idem triplum multiplicetur per omnes notas radicis & productum convenienter ſubſcribatur, nimirum pro monitu ipſarum literarum; fiatque diviſio & cætera, ut ſupra *Num.* 2. iterando hanc ipſam operationem quoties opus, uſque ad finem: ubi cum tandem nihil relinquitur, indicium eſt Cubi rationalis & radicis cub. rectè inventæ, cujus inſuper examen habebitur per geneſin, inventum ſcil. radicem rurſum cubice multiplicando.

Ratio

Ratio operationis patet ex Geneſi & adhibendâ Figurâ ſolidâ. Nam primam notam radicis exhibet cubus diagonalis b^3, quo ablato reſtat gnomon, conſtans ternis complementis b^2c [ſimilibus & æqualibus] itemque ternis bc^2; tum altero cubo diagonali c^3. Cum igitur prior complementorum ordo [b^2c] contineatur ſub Quadrato prioris notæ radicalis [b] & ſub ipſa altera nota c; ſequitur ſi horum complementorum aggregatum [$3b^2c$] dividatur per $3b^2$ [hoc eſt] per triplum radicis b ductum in radice b, ſeu quod idem eſt, per radicis Quadratum triplicatum, quod tunc in quoto emerget altera radicis nota c, &c.

§ Ex $a^3 - 3a^2b + 3ab^2 - b^3 \mid a - b \sqrt{c}$ quæſit.

$a^3 + 3a^2 + 3a$

mult. $-b + b^2$

$-3a^2b + 3ab^2 - b^3$

§ Ex $8c^3 - 12c^2d + 6cd^2 - d^3 \mid 2c - d \sqrt{c}$ quæſ.

$8c^3 + 12c^2 + 6c$

mult $(-d + d^2$

$-12c^2d + 6cd^2 - d^3$

Applicatio in Numeris.

Detur numerus 12167, cujus radix cubica 23, hæc ita diviſa intelligatur ut prima nota (2) pro valore loci ſui [10] conſtituat unam partem [20] altera vero [3] reliquam; hoc eſt radix 23 ſit diviſa in 20+3, quo ſuppoſito procedatur ut ſequitur.

ſubtr.

$$\begin{array}{rl} & 12167 \\ \text{subtr.} & \underline{8000} \end{array} \left\{ \begin{array}{l} b+c \\ 20+3 \end{array} \right.$$

$$\begin{array}{rr|l} & 4167 & \\ 3b & 60 & \\ \text{add.}\left\{\begin{array}{r} 3b^2 \\ 3bc \\ c^2 \end{array}\right. & \begin{array}{r} 1200 \\ 180 \\ 9 \end{array} & \begin{array}{l} 20=b \quad \overbrace{60=3b \quad 3=c}^{\text{mult.}} \mid 180=3bc \\ 400=b^2 \quad 1200=3b^2 \mid 9=c^2 \end{array} \\ \hline & 1389 & \text{totus Divisor} \\ & \text{mult. } 3 & \text{Quotus} \\ \text{subtr.} & 4167 & \end{array}$$

E X.

EXTRACTIO
Rad. Quadrato-quad.

[*Cujus nota eſt hæc*, $\sqrt{qq}$, vel w]

(1.) *Ex quantitatibus ſimplicibus.*

Quæ fit dividendo indicem poteſtatis propoſitæ per 4, & ſi Q. quadratum numericum præfixum ſit, ex eo ſeorſum w extrahendo.

Ex $\begin{cases} a^4 \\ b^4 \end{cases} \sqrt{qq}$. eſt $\begin{cases} a \\ b \end{cases}$ Ex $\begin{cases} b^4 \\ d^4 \end{cases} w$ eſt

(2.) *Ex Quantitatibus compoſitis.*

Ubi praxis ita ſe habet :

1.) Ex $b^4 + 4b^3c + 6b^2c^2 + 4c^3b + c^4 \mid b + c$ w quæſ.

$b^4 + b^3 + b^2 + b$ mult.

4 6 4

mult. $+4b^3 + 6b^2 + 4b$
$+ c + c^2 + c^3$

$+4b^3c + 6b^2c^2 + 4c^3b \mid + c^4$

1.) Ponatur w Quadrato-quadrati diagonalis b^4, quæ eſt b; & ipſum b^4 per ſubtractionem deleatur.

2.) Inventa

2.) Inventa radicis nota b; ejus Quadratum b^2; necnon Cubus b^3 ſubſcribantur ordine à dextra verſus ſiniſtram locis correſpondentibus, & multiplicetur, b quidem per 4, b^2 per 6, & b^3 rurſum per 4, ſemper; productum erit $4b^3+6b^2+4b$, conſtituens diviſorem pro ſequenti radicis nota, ex parte. Dividatur itaque $4b^3c$ per $4b^3$; Quotus c erit altera nota radicis: qua multiplicata in $6b^2$, ut fiat $6b^2c$; & ejus Quadrato c^2 in $4b$, ut fiat $4bc^2$; tum etiam in ſeipſam cubice ut fiat c^3, orietur totus diviſor $4b^3+6b^2c+4bc^2$: Qui demum totus multiplicetur in ſuum Quotum c, & productum ſubtrahatur ex ſupraſcripto. (Vel quod idem erit) acquiſitis terminis $4b^3+6b^2+4b$; multiplicetur $4b^3$ per c, $6b^2$ per c^2, & $4b$ per c^3, hoc eſt, terminus cubicus per ipſam notam radicalem c; terminus quadraticus per radicis Quadratum c^2; & terminus radicalis per radicis Cubum c^3; quibus denique jungatur ejuſdem Quadrato-quadratum c^4; eritque totum Collectum = ſuperiori.

3.) Si plures notæ Radicis, eruendæ reſtent, repetatur eadem operatio, *Num.* 2. aſſumendo omnes radicis notas prius inventas, &c. Ubi cum tandem nihil relinquitur, indicium eſt Quadrato-quadrati rationalis, & radicis recte inventæ.

Ratio operationis perinde ut in præcedentibus manifeſta fiet; niſi quod figuræ geometricæ ultra poteſtatem cubicam ſe non extendunt.

2.) Ex $x^4 - 4a^3b + 6a^2b^2 - 4b^3a + b^4 \mid a - b \sqrt{qq}.$

$x^4 + 4a^3 + 6a^2 + 4a$

$- b + b^2 - b^3$ mult.

$-4a^3b + 6a^2b^2 - 4b^3a \mid + b^4$

Praxis in Numeris.

Pari quoque methodo procedet, facilius tamen expedietur per geminam radicis Quadratæ extractionem, hoc eſt, Si ex numero propoſito primum extrahatur radix quadrata, & ex hac inventa radice iterum radix quadrata, hæc ipſa enim [ſecunda $\sqrt{q}$,] erit $\sqrt{qq}$. ipſius numeri biquadratici propoſiti.

Detur, exempli gratia, numerus 38416, hujus radix quadrata erit 196: Et hujus radix quadrata 14. Itaque 14 erit radix Quadrato-quadratica numeri propoſiti 38416.

Fuit hactenus (quantum hic loci ſufficere poteſt) de extractione Radicum ex poteſtatibus rationalibus, hoc eſt ubi requiſita praxis locum habet, & nihil relinquit.

Cæterum cum extractio prædicto modo fieri non poteſt, indicium eſt, quantitatem vel poteſtatem propoſitam eatenus eſſe irrationalem: cui proinde duntaxat ſignum radicis expetitæ prefigitur; & cum quantitas eſt compoſita ſignum iſtud per omnes terminos continuatur, & nomen Radicis Univerſalis accipit. Atque hæc eſt origo quantitatum Irrationalium ſive Surdarum, de quibus infra ſuo loco. Itaque, exempli gratiâ,

Radix

Radix Quadr. $\left\{\begin{matrix} a^2 \\ ab \\ 5b^2c \\ 3c^4 \end{matrix}\right\}$ erit $\begin{matrix} \sqrt{a^2} \\ \sqrt{ab} \\ \sqrt{5b^2c} \\ \sqrt{3c^4} \end{matrix}$ [Quoniam index [3] non potest exactè dividi per 2.

[Hoc loco quidem index [4] potest dividi per 2, sed numerus præfixus 3. non est Quadratus

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Radix Cubica $\left\{\begin{matrix} a^4 \\ 8a^2b \\ 6c^3 \\ 5d^2 \end{matrix}\right\}$ erit $\begin{matrix} \sqrt{c.} \ a^4 \\ \sqrt{c.} \ 8a^2b \\ \sqrt{c.} \ 6c^3 \\ \sqrt{c.} \ 5d^2 \end{matrix}$

Item Radix Quadr. universalis $\left\{\begin{matrix} a^2+b^2 \\ a^2+c^2-b^2 \\ 4b^2+bc^2 \end{matrix}\right\}$ erit $\begin{matrix} \sqrt{a^2+b^2} \\ \sqrt{a^2+c^2-b^2} \\ \sqrt{4b^2+bc^2} \end{matrix}$

* Hoc est addantur duo Quadrata, a^2 & b^2, & ex aggregato extrahatur radix quadrata.

Rad. Cubica universalis $\left\{\begin{matrix} a^3+8b^3 \\ 2a^3-b^3 \\ b^2c+2bc^2-c^3 \end{matrix}\right\}$ erit $\begin{matrix} \sqrt{c.a^3+8b^3} \\ \sqrt{c.2a^3-b^3} \\ \sqrt{c.b^2c+2bc^2-c^3} \end{matrix}$

II. Logistica FRACTIONUM.

IN QUA

Uberioris exercitii gratia, conjungemus Praxin Numerosam cum Speciosa.

Cap. I.

Notatio Fractionum.

§. *Fractiones propriæ.*

Numerator,	1	2	3	a	$2a$	a^2	}	Suppositio b majori quam a.
Denominator,	2	3	5	b	$3b$	b^2	}	

§. *Fractiones impropriæ.*

$\frac{2}{2}$	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{12}{5}$	$\frac{a}{a}$	$\frac{b}{a}$	$\frac{3b}{2a}$	$\frac{2b^2}{a^2}$	Vid. & *Cap.* II. *Memb.* 1.

Ubi *Not.* Cum Denominator & Numerator sunt =, Fractio semper est = 1, &c.

§. *Fractione*

§. *Fractiones mixtæ cum integris.*

$3\frac{1}{2}$ | $5\frac{3}{4}$ | $8\frac{5}{8}$ || $a+\frac{a}{b}$ | $2b-\frac{c^2}{3a}$

§. *Fractiones fractionum, ſeu Fractiones ſecundæ.*

$\frac{1}{2}$ ex $\frac{1}{4}$ hoc eſt, dimidium ex tribus quartis. Quod etiam ſcribi ſolet, $\frac{1}{2}$, $\frac{1}{4}$, interjecto nimirum commate, item $\frac{2}{3}$ ex $\frac{4}{5}$, duæ tertiæ ex quatuor quintis.

In his, poſterior fractio ſeu terminus, eſt principalis, & major, qui refertur ad ipſum totum: Sed prior terminus, qui eſt minor, refertur ad poſteriorem.

Sic in literis: $\frac{a}{b}$ ex $\frac{2a}{3b}$ Sitque $a=3$. $b=4$.

CAP. II.

Reductio ſeu Tranſmutatio Fractionum.

§. 1. MEMBRUM.

Fractiones impropriæ ad integra reducere.

Dividiatur Numerator per Denominatorem, Quotus dabit integra, &c. Reducantur.

ex. gr. 1.) $\frac{12}{4}$ | 3 integra quotus. *Sint Fractiones propoſitæ:*

2.) $\frac{8}{3}$ | $2\frac{2}{3}$ quotus. *Erunt reductæ.*

3.) $\frac{16}{7}$ | 8 quotus.

4.) $\frac{65}{12}$ | $5\frac{5}{12}$ quotus.

Sic in literis: $\frac{2aa}{a}$ | $2a$ quotus | $\frac{6bc}{2c}$ | $3b$ quot. $\left|\frac{a}{a}\right|$ 1

§. 2. Membrum.

Integra in formam Fractionis redigere.

1.) Si integra sint sola, nec detur certus denominator, subscribatur proposito numero vel quantitati unitas, & factum erit.

Dentur quantitat. Integr.	*Orientur Fractiones.*
6	$\frac{6}{1}$
$1\frac{5}{1}$	$\frac{11}{1}$
1	$\frac{1}{1}$
a	$\frac{a}{1}$
$2b$	$\frac{2b}{1}$
$3ab$	$\frac{3ab}{1}$

2.) Si integra sint sola, & detur certus denominator Fractionis optatæ.

Multiplicitur quantitas proposita per datum denominatorem, & huic deinde subscribatur producta.

Sit numerus 7 redigendus in fractionem, cujus denominator, sit 4, fient $\frac{28}{4}$.

[Quod

[Quod quidem nihil aliud eſt ac ſi dicam, ſeptem ulnas vel libras integras, reſolvendas eſſe in Quadrantes.]

Detur quantitas b, & denominator fractionis $a-b$, orietur fractio impropria $\frac{ab-bb}{a-b}$

Detur quantitas $2a+c$, & denominator f, orietur fractio $\frac{2af+cf}{f}$

3.) Si integra habeant annexam fractionem, multiplicetur quantitas integra per denominatorem fractionis adhærentis; producto addatur numerator, & ſubſcribatur aggregato denominator datus.

Dentur,	$2\frac{1}{4}$	$3\frac{2}{5}$	$9\frac{4}{10}$	$12\frac{6}{25}$
	$\frac{8}{3}$	$\frac{11}{2}$	$\frac{22}{4}$	$\frac{312}{25}$
Fiunt,	$\frac{11}{4}$	$\frac{17}{5}$	$\frac{24}{10}$	$\frac{306}{25}$

Detur $c+\frac{de}{a}$ prodibit $\frac{ac+de}{a}$

Detur $b+c+\frac{b^2+c^2}{b-c}$ prodibit $\frac{2b^2}{b-c}$

Detur $2ab-\frac{a^2c}{b}$ prodibit $\frac{2abb-a^2c}{b}$

§. 3. Membrum.

Fractiones in terminis majoribus propoſitas, ad minores terminos æquivalentes (ſi fieri poſſit) reducere.

Poteſt enim quælibet fractio, una eademque in numeris modis exprimi, inter quos notiſſimus ille eſt qui terminos habet minimos.

Interim non omnis fractio ad minores terminos reducibilis eſt, ſed illæ duntaxat, in quibus numerator & denominator communem aliquam diviſorem, vel etiam plures, agnoſcant.

Si igitur quæratur utriuſque termini (hoc eſt numeratoris & denominatoris) communis menſura maxima (ut infra docebitur) & per hanc uterque dividatur, quoti dabunt terminos correſpondentes minimos,

Vel quod idem erit, fiant utrobique diviſiones continuæ, per eoſdem diviſores quoſcunque, qui ſe offerunt, ac nihil relinquunt, uſque dum nullus amplius communis diviſor occurrat niſi unitas.

Itaque in ſpecie (potiſſimum quoad Praxin Numericam) hoc loco obſervantur caſus ſequentes.

1.) Cum denominator poteſt dividi exacte per numeratorem, tunc ipſe numerator eſt communis diviſor maximus. Et hoc in caſu, numerator fractionis reductæ ſemper eſt unitas.

Dentur Fractiones.

$\frac{6}{24}$ $\frac{5}{15}$ $\frac{8}{40}$ $\frac{4}{32}$ $\frac{7}{63}$

Prodibunt his æquivalentes in terminis minimis.

$\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{8}$ $\frac{1}{9}$

Quod ſi autem illud ipſum non ſtatim appareat, cum ſcil. numeri ſunt majores, tunc procedendum erit juxta ſequentia.

2.) Si

2.) Si terminus uterque ſit numerus par, bipartire eos quouſque licet.

Detur fractio $\frac{32}{64}$ per continuam dimidiationem orienter hæ, $\frac{16}{32} \mid \frac{8}{16} \mid \frac{4}{8} \mid \frac{2}{4} \mid \frac{1}{2}$ quarum ſingulæ æquivalent fractioni datæ; ſed ex his $\frac{1}{2}$ tanquam terminis minimis conſtans, eſt notiſſima.

Item detur fractio $\frac{16}{24}$; per dimidiationem prodibunt huic æquivalentes, $\frac{8}{12}$ $\frac{4}{6}$ $\frac{2}{3}$.

3.) Si uterque terminus habeat ad dextram unam vel plures nullitates, deleantur utrobique numero pares [quod nihil aliud eſt quam per 10, vel 100, &c. utrinque dividere.]

Dentur $\frac{30}{40}$ vel $\frac{3000}{4000}$ vel $\frac{300000}{400000}$

abjectis utrinque totidem nullitatibus fiunt $\frac{3}{4}$

Dentur $\frac{7000}{4000}$ prodeunt primum $\frac{7}{4}$ &

porro dimidiando $\frac{3}{2}$

4.) Si termini aliter ſe habeant, nec tamen numeri ſint valde magni, facile etiam apparebit [ex tabula Pythagorica] ſi aliquem vel aliquos communes diviſores admittant, cum quibus proinde operatio inſtituatur ut dictum.

Detur fractio $\frac{42}{63}$; hic primum uterque terminus poteſt dividi per 7, unde prodeunt $\frac{6}{9}$, & hic porro uterque poteſt dividi per 3, ut prodeant $\frac{2}{3}$.

Item

Item detur fractio $\frac{15}{35}$; dividendo utrobique per 5, prodibunt $\frac{3}{7}$.

Item detur fractio $\frac{48}{54}$; dividendo per 6 habebitur æquivalens $\frac{8}{9}$.

5.) Si dictis modis res non succedat, puta cum numeri sunt majores, tunc demum communis mensura maxima inquiratur hoc modo.

Dividatur semper terminus major per minorem, tam in residuis quam ipsis datis, hoc est, semper divisor præcedens per suum residuum, nulla habitâ ratione quotorum: Ultimus divisor, qui nihil relinquit, erit utriusque communis mensura maxima. [*Vid. Exemplum* A.]

Quodsi autem in hujusmodi divisione tandem remaneat unitas, indicium est, terminos fractionis propositæ nullum communem divisorem agnoscere præter unitatem, adeoque fractionem istam ad minores terminos reduci non posse. [*Vid. Exemplum* B.]

(12 resid.

A] Detur fractio $\frac{105}{432}$ | Div. ~~432~~ 1) ~~105~~ | 4

(9 2) ~~105~~ ~~12~~ | 8 | (3 3) ~~12~~ ~~9~~ | 1 | 4) 9 3 | 3 remanet nihil.

& ultimus divisor erat 3, hic igitur est communis divisor maximus pro reducenda fractione data, ut sequitur.

Numerator.

Numerator.		*Denominator.*		*Fractio reducta.*
Div. ~~105~~ ~~3~~	35	~~432~~ ~~3~~	144	$\frac{35}{144}$

B] Proponatur autem porro hæc ipſa fractio acquiſita $\frac{35}{144}$, puta ad minores terminos reducendo.

Div. 1.) ~~14~~ (4 ~~35~~	4	2.) (3 ~~35~~ ~~4~~	8	3.) ~~4~~ ~~3~~	(1 rem. un. 1

unde colligitur, fractionem hanc $\frac{35}{124}$ jam in minimis ſuis terminis ſtare, nec ulteriorem reductionem admittere.

§. In Praxi Specioſa, ſi quantitates non ſint nimis compoſitæ, communis diviſor * facile agnoſcitur, [dummodo quis in praxi Multiplicationis & Diviſionis ſatis ſit exercitatus] quo habito, eodem modo proceditur ut ſupra in numeris, dividendo ſcilicet utrumque terminum per communem diviſorem maximum repertum, & aſſumendo quotos loco propoſitorum.

* *Si quem habet.*

Cum autem quantitates valde ſunt compoſitæ ac prolixæ, inquiſitio communis diviſoris operoſior eſt & tædioſior, quam ut hoc loco [inter elementa] tradi & tractari poſſit.

Sufficient igitur quæ ſequuntur exempla.

Fractiones

Fractiones propositæ.	*Communes Divisores.*	*Fractiones reductæ.*
$\frac{abc}{bd}$	b	$\frac{ac}{d}$
$\frac{4a^2b^2}{2abc}$	$2ab$	$\frac{2ab}{c}$
$\frac{abd^2}{b^2d}$	bd	$\frac{ad}{b}$
$\frac{2a^3b^4c}{6ab^2c}$	$2ab^2c$	$\frac{a^2b^2}{3}$
$\frac{b^2c+b^2d}{cd+d^2}$	$c+d$	$\frac{b^2}{d}$
$\frac{a^2+2ab+b^2}{a^2-b^2}$	$a+b$	$\frac{a+b}{a-b}$
$\frac{f^2c-f^2d-c^3+c^2d}{dc-d^2}$	$c-d$	$\frac{f^2-c^2}{d}$
$\frac{c^2-d^2}{bc+bd+c^2+cd}$	$c+d$	$\frac{c-d}{b+c}$
$\frac{a^2+b^2}{a^2-b^2}$	$a+b$	$\frac{a^2+b^2-ab}{a-b}$

D E

DE

Æquationum Algebraicarum

Inventione, Diſtinctione, Reductione, Reſolutione.

QUotieſcunque Problema aliquod ſive quæſtio proponitur, putetur præſtitum eſſe quod postulatur: Et pro quantitate quæſita (ſive ſit numerus ſive magnitudo) ponatur x, y, vel z, hoc eſt, una ex ultimis literis Alphabeti; pro datis autem priores, ut a, b, c, &c. quo facilius data à quæſitis dignoſcantur.

Deinde Quantitates tam datæ quam quæſitæ ſecundum conditionem quæſtioni convenientem tractentur atque comparentur, addendo, ſubtrahendo, multiplicando, dividendo, donec tandem aliquid inveniatur, quantitati de qua quæritur, vel ſuæ ad quam adſcendet, poteſtati æquale.

Quoniam autem in omni fere æquatione, ubi primum ex involucris quæſtionis emergit, nota cum ignotis miſcentur ac confunduntur: Ideo termini cujuſque æquationis ita ſunt ordinandi ac reducendi ut data faciant unam partem, & quæſita alteram. Tum vero Quantitates utrinque ad formam lateris ſeu radicis redigendæ ſunt. Id quod reductionem æquationum ad ſimpliciſſimam formam vocamus; quæ his potiſſimum axiomatis innititur.

Si

Si æqualibus æqualia addantur vel ſubducantur, aggregata ac reſidua erunt æqualia.

Si æqualia per æqualia multiplicentur vel dividantur, quæ oriuntur erunt æqualia.

Æqualium radices homogeneæ, erunt æquales.

Porro æquationes non ſunt unius generis, ſed tot earum exiſtunt claſſes, quot gradus poteſtatum ſeu dimenſionum in ſcala Quantitatum progreſſionali. Unde oriuntur æquationes,

1. *Simplices*, } } 3. *Cubicæ*,
2. *Quadraticæ*, } } 4. *Quadrato-quadr.* &c.

Quarum ſingulæ ſingulas quaſi Algebræ partes conſtituunt.

Reductio Æquationum (potiſſimum ſimplicium ſeu primarum) ad ſimpliciſſimam formam perficitur,

1. *Additione.*

Sit $x - a = b + c$
add: $+a \quad +a$
erit $x = b + c + a$

$Q - b = 0$
$+b \quad +b$
$Q \quad = b$

$c - x = 0$
$+x \quad +x$
$0 = x$

2. *Subtractione*

2. *Subtractione.*

Sit Q. $+x=b+c$ — $x+a-b=2x+d$
subt. $+x$ $+a$ — x subtr.
erit Q. $=b+c-a$ — $a-b=x+d$
subt. $+d$ $+d$
$a-b-d=x$

[*Nota:* Additionis itaque & Subtractionis hujus compendium erit Transpositio Quantitatum ex una parte æquationis in alteram facienda semper sub signo contrario.]

Sit $a-b-x=b+x$
erit $a-2b=2x$

3. *Multiplicatione.*

Sit $\begin{pmatrix} x = b \\ a \\ a \text{ mult. } a \end{pmatrix}$ — $\frac{Q}{3b}=a-c$

erit $\frac{ax}{a}=ab$ — mult. per $3b$

hoc est $x=ab$ — erit $Q=3ab-3bc$

4. *Divisione.*

Sit $x^2=6x$ — $3x^2=9x$
x div. — $3x$ div.
erit $x=6$ — $x=3$

Sit

Sit $x^2 + ax = bx + cx$
x div.
erit $x + a = b + c$
& $x = b + c - a$

5. *Radicum extractione.*

Sit $x^2 = a$
extr. $\sqrt{}$
$x = a$

Sit $y^2 = ab$
extr. $\sqrt{}$
$y = \sqrt{ab}$

Sit $x^2 + ax + \frac{1}{4}a^2 = b^2$
extr. $\sqrt{}$
$x + \frac{1}{2}a = b$
& $x = -\frac{1}{2}a + b$

Sit $x^3 - 3bx^2 + 3b^2x - b^3 = c^3$
extr. $\sqrt{c}$
$x - b = c$
& $x = b + c$

SEQUUNTUR

Problemata *ſeu* Quæſtiones:

Sub genere Æquationum ſimplicium.

1. INvenire numerum, quo multiplicato per 3, ex producto ſubtractis 5, reſiduoque diviſo per 2, ſi quoto huic addatur ipſe numerus quæſitus aggregatum ſit 40.

Ponatur numerus quæſitus $=x$; eritque

1. *Praxis Numeroſa.*

Num. quæſitus $=x$
mult. per 3
product. erit $3x$
hinc ſubtr. 5
reſid. erit $3x-5$
quod div. per 2
quotus erit $\frac{3x-5}{2}$
huic add. num. quæſitus
aggregatum erit $\frac{3x-5}{2}+x=40$
quare multiplicando erit $3x-5+2x=80$
hoc eſt $5x-5=80$
& tranſponendo $5x=80+5$
hoc eſt $5x=85$
& dividendo per 5
$x=17$ = numero quæſito.

§. *Examen.*

Numerus inventus 17
multipl. per 3
51
ſubtr. 5
Divid. per 46
2
23
add. 17
provenit 40 quod erat propoſitum.

2. *Praxis Specioſa.*

Sit 3 ſeu numerus multiplicans $=a$ } Et hi omnes
5 ſubtrahendus $=b$ } erunt dati
2 diviſor $=c$ } vel aſſumti
40 aggregatus $=d$ } prolubitu.

Jam igitur: Num. quæſitus $= x$
mult. per a
product. ax
ſubtr. b
reſid. $ax=b$
divid. per c
quotus $\frac{ax}{c}=\frac{b}{c}$
add. x
aggregat. $\frac{ax}{c}+x=\frac{b}{c}+d$

ergo multiplicando: $ax-b+cx=cd$
& tranſponendo: $ax+cx=cd+b$
& dividendo per $a+c$
$x=\frac{cd+b}{a+c}$

Hinc

Hinc regula pro hujuscemodi Quæstionibus generalis: Numerus æquationis datus [ut h. 1.40.] multiplicetur per datum divisorem; Producto addatur numerus subtrahendus; Totum hoc dividatur per multiplicantem auctum divisore; Quotus dabit numerum quæsitum.

§. Ubi observandum, quod inventio quæsiti perficiatur operatione seu methodo ipsi quæstioni contraria. Et sic quoque in cæteris.

2. Invenire numerum, quo multiplicato per 12, & ad productum additis 48, tantundem proveniat, ac si idem numerus quæsitus per 18 esset multiplicatus.

Vel generaliter ita,

Invenire numerum, quo multiplicato per *a*, & ad productum addito *b*, tantundem proveniat, ac si idem numerus esset multiplicatus per *c*.

3. Invenire numerum, cui si addantur 11, & ab eodem [numero primo] si subtrahantur 7, summa additionis sit dupla residui subtractionis.

Vel generaliter.

Invenire numerum, cui si addatur *b*, & ab eodem [numero primo] si subtrahatur *c*, summa additionis sit ad residuum subtractionis in ratione data, puta dupla, tripla, quadrupla, &c.

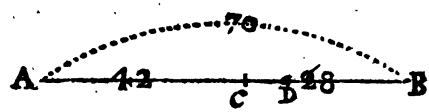
4. Reperire numerum, cui si addatur ipsius duplum [triplum, quadruplum, &c.] proveniat ejusdem numeri quadratum.

5. Invenire numerum qui ſi ad ſeipſum addatur; ſumma per eundem multiplicetur; ex producto idem numerus ſubtrahatur; & tandem reſiduum hoc per eundem dividatur, ut proveniant 13.

6. Dividere numerum 16 in duas partes, ita ut quadratum majoris partis excedat quadratum minoris per 32.

7. Dividere numerum 36 in duas partes, ita ut ſi ad primam addantur 12, & ad alteram 6, ſumma prior ſit dupla poſterioris.

8. Sit linea AB (partium 70) ſecta utcunque in C [ita ut AC ſit 42, BC 28] oportet eandem lineam AB aliter ſecare, v. gr. in D, ita ut rectangulum ADC ſit æquale quadrato DB quæritur interſegmentum CD [quo habito innoteſcent AD, DB].

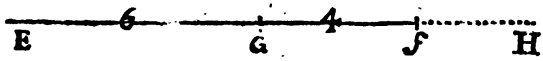


§. *Vel ita in Numeris.*

Sint duo numeri 42 & 28; inveniendus eſt tertius, quo addito ad 42, ſi totum hoc multiplicetur per ipſum numerum quæſitum, tantundem proveniat, ac ſi eodem numero quæſito ſubtracto ex 28. reſiduum multiplicetur quadratè.

9. Sic

9. Sit linea EF, ſecta utcunque in G [ita ut EG ſit 6, GF 4] oporteat hanc rectam EF producere [v. gr. in H] ita ut rectangulum EHF ſit æquale quadrato GH; quæritur prolongatio FH.



§. *Vel ita in Numeris.*

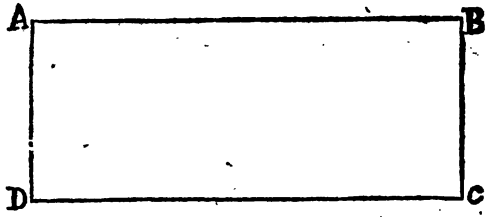
Sint duo numeri 6 & 4; inveniendus eſt tertius, quo addito ad ſummam utriuſque datorum, ſi totum hoc multiplicetur per quæſitum, tantundem proveniat, ac ſi aggregatum ex quæſito & minore datorum multiplicetur quadratè.

10. Dux bellicus dum exercitum ſuum in aciem quadratam diſponit, ſuperſunt ipſi milites 284; at cum ſingulas lineas uno milite auget [ſeu cum latus quadrati unitate majus conſtituit] deſunt ipſi ad explendum quadratum 25 milites; quæritur quot milites habuerit.

11. Capitaneus quidam emittit $\frac{1}{3}$ ſuorum militum + 10, reſtant ipſi $\frac{1}{2}$ + 15. Quær. quot mil. habuerit.

12. Eſt exercitus, cui ſi addatur ſui $\frac{1}{2}$, $\frac{2}{3}$, & $\frac{3}{4}$ —5000, ſumma erit 100000. Quær. numerus iſtius exerc.

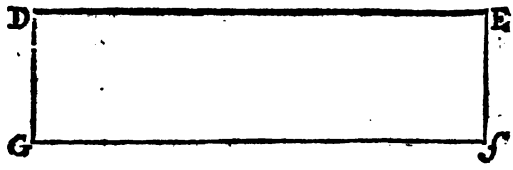
13. In rectangulo ABCD, differentia lateris majoris AB, & lateris minoris BC, est 12, differentia autem quadratorum à dictis lateribus, est 1680. Quæruntur latera rectanguli ABCD.



Hoc est, proponendi generaliter.

Datâ in rectangulo differentiâ laterum, & differentiâ quadratorum laterum, invenire latera rectanguli.

14. Rectanguli DEFG longitudo DE est dupla latitudinis EF; & summa quadratorum longitudinis ac latitudinis est decupla summæ ipsorum laterum DE, EF. Quæruntur latera rectang. DEFG.



Seu generaliter.

Data in rectangulo ratione laterum, & ratione summæ quadratorum laterum, ad summam ipsorum laterum, invenire latera rectanguli.

15. Inve-

15. Invenire duos numeros in ratione, ut 2 ad 3 quorum productum, si invicem multiplicentur; sit 54.

Sit minor numerus = X

$\therefore 2 \dots 3 : X \mid \frac{3x}{2}$ = majori numero.

vel, sit communis factor [quo in terminos datæ rationis ducto proveniant ipsi numeri desiderati] = X : ∴ numerorum quæsitorum minor = 2X & major = 3X

16. Invenire duos numeros, quorum ratio ad invicem sit ut 4 ad 5 : & summa quadratorum à singulis, sit 2624.

17. Invenire latus quadrati, cujus area sit ad summam laterum in data ratione, puta ut 45 ad 12.

18. Invenire latus cubi, cujus superficies ad soliditatem sit in data ratione, puta ut 6 ad 11.

19. Quidam conducit operarium, ea conditione ut singulis diebus quibus operi institerit, accipiat 12 Stufros, sed quibus otiatus fuerit, mulctetur 8 Stufris. Elapsis autem diebus 390 neuter alteri quicquam debet. Quæritur quot diebus fuerit operatus, quot otiatus. 3 *s.* 2 *s.*

20. Dominus quidam conducit Servum, cui pro mercede annuâ pollicetur 24 Florenos, unà cum pallio. Finitis autem 8 mensi- 24 *l.*

bus Servus impetrato diſceſſu, mercedis loco accepit pallium + 13 Flor. Quæritur quanti conſtiterit pallium.

13 *l.*

21. Quidam interrogatus quot annos haberet, reſpondit, Si annorum meorum $\frac{2}{3}$ quadruplicem, & producto addam $\frac{1}{2}$ eorundem + 50; ſumma in tantum ſupra 100 excreſcet, quantum nunc infra 100 ſubſiſtit annorum meorum numerus.

22. Rogatus quidam quota diei hora eſſet, reſpondit, Dies hoc tempore eſt horarum 16; ſi jam $\frac{1}{2}$ horarum elapſarum addantur $\frac{2}{3}$ reſiduarum, prodibit hora quæſita ab ortu Solis numeranda.

23. A Norimberga ad Romam ſunt 140 milliaria: eodem tempore ex ſingulis his urbibus exit viator, quorum unus quolibet die confecit milliaria 8, alter 6. Quæritur quot ab initio itineris diebus elapſis ſibi obviam venient, & quot milliaria ſinguli perfecerint.

24. Tabellarius aliquis conficit ſingulis diebus milliaria 6: poſt 8 dies dies inſequitur hunc alius, qui ſingulis diebus conficit 10 milliaria. Quæritur quoto die priorem aſſequetur.

25. Quidam Tabellarius conficit ſingulis diebus milliaria 6: poſtquam vero 56 milliaria perfecit, inſequitur eum alter, qui ſingulis diebus 8 conficit. Quæritur quot diebus hic illum aſſequetur.

26. Qui-

26. Quidam coemit tres libros, in ratione pretii, ut 12, 5, 1: Si pretium primi duplicetur, ſecundi triplicetur, tertii quadruplicetur; ſumma horum productorum in tantum ſuperabit 10 imperiales, quantum ſumma pretium maximi & medii eſt infra 5 imperiales. Quær. quanti conſtiterint dicti libri.

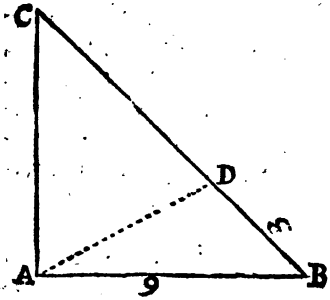
27. Sit numerus 50 ſecundus in duas partes, ita ut parte majore divisâ per 7, & minore multiplicatâ per 3, ſumma hujus producti & prioris quoti reſtituat eundem numerum propoſitum 50.

28. Sit numerus 20 ſecundus in duas partes, ita ut Quadratum minoris partis ſubductum ex Quadrato majoris relinquat ipſum numerum propoſitum 20 [vel, relinquat duplum, triplum, &c. numeri propoſiti.

29. Si quis ſingulis ſeptimanis lucretur 30 imperiales. Quæritur quantum qualibet ſeptimanâ expendere debeat, ut ſibi ad finem anni ſuperſint 500 Imperiales, unà cum expenſa 4 ſeptimanarum.

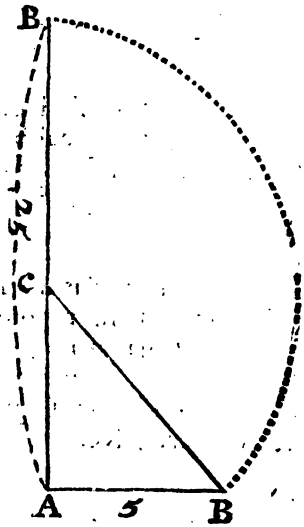
30. Operarius quidam exactis 40 ab opere incepto ſeptimanis numerat in repoſito 28 Imperiales — mercede 3 ſeptimanarum; deprehenditque ſe expendiſſe 36 Imp. + mercede 11 ſeptimanarum. Quæritur quantum ſingulis ſeptimanis pro mercede acceperit, &c.

31. In



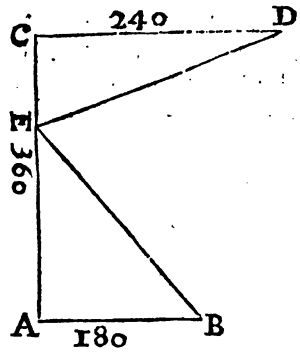
31. In Triangulo rectangulo ABC, datur basis AB=9, & differentia reliquorum laterum, hoc est, Segmentum BD,=3. Quæruntur latera AC, BC.

32. In Triangulo rectangulo ABC, datur basis AB=5, & summa reliquorum laterum AC+BC=25. Quæruntur ipsa latera AC, BC, distinctim.



33. Sint

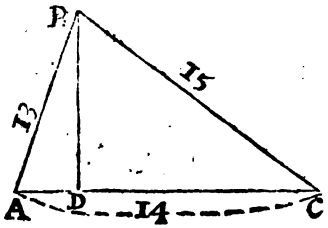
33. Sint duæ turtes, AB altitudinis 180 pedum, & CD 240, in distantia AC 360 pedum. Applicare oporteat scalam super linea AC, puta in E, ejus longitudinis, ut inde ad summitatem utriusvis turris æque pertingat. Quæritur punctum E in linea distantiæ, ut & longitudo scalæ EB, ED.



§. *Hoc est, proponendo abstracte.*

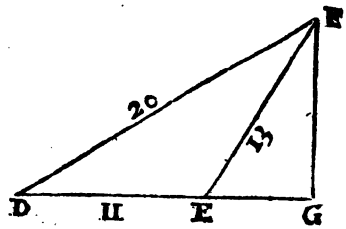
Duorum Triangulorum rectangulorum, æquales hypotenusas habentium, datur summa basium & cujusque altitudo. Quæruntur singulæ bases cum communi hypotenusâ.

34. In Triangulo ABC, dantur singula latera, AB=13, AC=14, BC=15; ductâque perpendiculari BD. Quæruntur segmenta baseos, AD, DC.

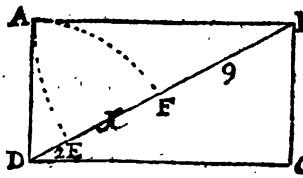


35. In

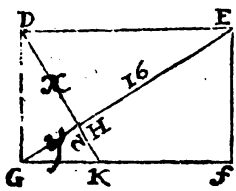
35. In triangulo obtusangulo DEF dantur singula latera, utpote DE 11, EF 13, DF 20; demissoque in basin prolongatam perpendiculo FG. Quæritur baseos prolongatio EG.



36. In rectangulo ABCD, datur differentia inter longitudinem AB, & diagonalem BD, hoc est DE=2; itemque differentia inter latitudinem AD & diagonalem BD, hoc est FB= 9. Quæruntur rectanguli latera AB, AD.

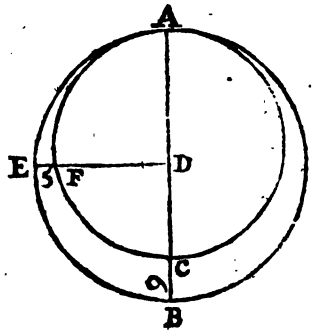


37. In rectangulo DEFG, ex angulo D ad latus oppositum GF ducta est recta DK, intersecans diagonium EG ad angulos rectos in H: daturque segmentum HK=2, & HE= 16. Quæruntur latera rectanguli.



38. Sit

38. Sit circulus, cujus diameter A B, quem alius minor, cujus diameter AC, interius tangat in A: ereſtoque ex D [centro majoris] ſuper diametro AB, perpendiculariter radio DE, qui ſecet peripheriam minoris in F; detur BC [differentia diametrorum] =9, cum ſegmento EF = 5. Quæruntur circulorum dictorum Diametri AB, AC.



39. Duo ſodales habent aliquot aureos: quorum A dicit ad B; Si dederis mihi 1 aureum de tuis, totidem habebo ac tibi reſtabunt. Imo, reſpondet B, ſi tu mihi dederis 1 aureum de tuis, duplum habebo eorum qui tibi reſtabunt. Quæritur quot aureos quiſque habuerit.

40. Quidam emit duos equos, cum Ephippio conſtante 100 Florenis: quod [Ephippium] ſi imponatur primo Equo [A] uterque equorum æquali pretii cenſebitur: ſin vero imponatur alteri equo [B] dictum Ephippium, hic duplo erit pretioſior ptiori. Quæritur quanti conſtiterint dicti equi.

41. Oeno-

41. Oenopola quidam habet duplex vinum v. gr. A & B: quæ si misceantur ad partes æquales, Cantharus mixti constat 15 Stufris; si vero misceantur in ratione sesquialtera, puta si toties sumantur 2 Canthari de A, quoties 3 de B, Cantharus constabit 14 Stufris. Quæritur de pretio cujusque vini sigillatim.

42. Interroganti filio quot annos haberet, respondit pater, Si annis meis detraxeris 5, & reliquum diviseris per 8, quotus erit $\frac{1}{3}$ annorum tuorum: si vero annis tuis addideris 2, & totum multiplicaveris per 3, detractis ex producto 7, habebis ipsum numerum annorum meorum. Quæritur de ætate tam patris quam filii.

43. Reperire duos numeros, quorum summæ si addantur 6, totum sit duplum majoris numeri; & quorum differentiæ si detrahantur 2, quod remanet sit dimidium minoris.

44. Reperire duos numeros quorum productum sit 240, & triplum majoris divisum per minorem sit 5.

45. Duo volunt emere domum pro 1200 Florenis; dicitque A ad B, Si dederis mihi $\frac{2}{3}$ tui argenti, ego solus potero domum hanc redimere: cui B, si a te acceperim $\frac{3}{4}$ tuæ summæ, & ego solus redemero domum propositam. Quæritur quantum argenti quisque habuerit.

46. Juvenes

46. Juvenes aliquot & virgines debent hospiti pro exhibito convivio, 37 Florenos, ea ratione ut ut juvenum cuique assignentur 3 Floreni, virgini autem 2. Sique fuissent tot juvenes, quot fuerunt virgines, & e contra, servatâ eadem conditione, debuissent 4 Florenis minus quam nunc. Quæritur numerus juvenum ac virginum.

47. Dux bellicus post commissum prælium lustrans exercitum, cujus peditatus fuerat triplus ipsius equitatus, deprehendit ante prælium aufugisse ex peditatu $\frac{1}{12} - 120$, ex equitatu vero $\frac{1}{20} + 120$, tum $\frac{1}{4}$ totius exercitus in præsidia distributos esse [connumeratis infirmis & vulneratis] ac supersesse sibi $\frac{1}{8}$ totius exercitus; reliquis qui desiderantur, ipso prælio extinctis aut captis; quorum [extinctorum] numero si addantur 3000, summa erit = dimidio peditum quos initio habuit. Quæruntur numeri singulorum.

48. Dividere numerum 100 bis in duas partes, ita ut major pars primæ divisionis sit tripla minoris partis secundæ divisionis; & major secundæ sit dupla minoris primæ.

49. Dividere numerum 30 bis in duas partes, ita ut major pars primæ divisionis cum minori secundæ sit 33; & summa partium minorum subducta ex summa majorum relinquat 14.

50. Maritus uxor & filius habent annos 96, ita ut mariti anni & filii simul faciant uxoris + 15; sed uxoris cum filii faciant annos mariti + 2. Quæritur ætas singulorum.

51. Tres

51. Tres Mercatores ex diverſis nundinis in diverſorio convenientes, connumerant lucra ſua, habentque ſummam 780 Flor. Porro ſi addatur lucrum primi & ſecundi, & ex aggregato ſubtrahatur lucrum tertii, remanet lucrum primi + 82 Flor. ſin autem addatur lucrum ſecundi & tertii, & ex aggregato ſubtrahatur lucrum primi, remanet lucrum tertii — 43 Flor. Quæritur de lucro ſingulorum.

52. Tres perſonæ, A, B, C, debent certam pecuniæ ſummam, ita ut A cum B ſimul debeant 210 Florenos; B cum C, 290, & C cum A 400. Quæritur quantum unuſquiſque debeat.

53. Invenire tres numeros, ita ut primus cum dimidio reliquorum, ſecundus cum $\frac{1}{3}$ reliquorum & tertius cum $\frac{1}{4}$ reliquorum ſemper faciant 34.

54. Detur Quadratum in 9 quadratula diſtinctum: oportet invenire ac diſponere numeros per ſingulas areolas, ita ut ſumma cujuſque ordinis ternarii, ſive lateraliter ſive diagonaliter connumerando, ſemper ſit 15.

55. [Theorema] Si dentur numeri quotcunque & qualeſcunque, fiatque ſubtractio minoris cujuſque ex proxime majori; Dico ſummam iſtarum differentiarum æquari differentiæ numerorum maximi & minimi.

DE

DE
Æquationib. Quadr.
Earumque Solutione.

ÆQuationum Quadraticarum tres numerantur Caſus ſeu Formulæ, ut ſequitur:

Primus CASUS.

$$x^2 = ax + b^2$$

Solutio $\frac{1}{2}a$ mult.

$$\frac{\frac{1}{2}a}{\frac{1}{4}a^2 + b^2}$$

$$x = \sqrt{\tfrac{1}{4}a^2 + b^2} + \tfrac{1}{2}a = BD$$

Examen ſeu reſtitutio.

$$x - \tfrac{1}{2}a = \sqrt{\tfrac{1}{4}a^2 + b^2}$$

$$x - \tfrac{1}{2}a$$

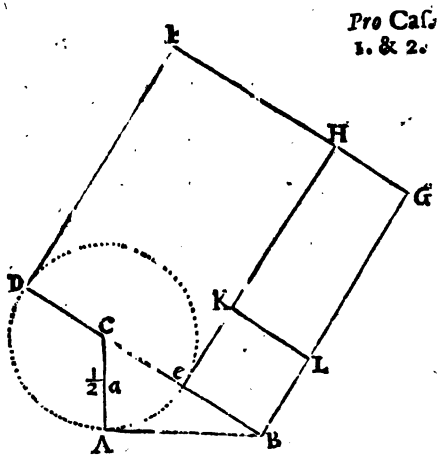
$$x^2 - ax + \tfrac{1}{4}a^2 = \tfrac{1}{4}a^2 + b^2$$

$$\therefore x^2 = ax + b^2$$

Demonstratio Primi Casus.

Sit AC=$\frac{1}{2}a$, & AB=b: quibus conjunctis ad angulum rectum in A, describatur centro C, radio CA, Circulus ADe, ductâq; recta DCB=x, fiat super eadem Quadratum BDFG=x^2; agatur porro eH, parallela ipsi BG.

Pro Casu 1. & 2.



Dico 1.) x^2 esse $=ax+b^2$. Nam primum DCe=a, & DF=x, componunt rectang. DH=ax; deinde per 36.111 rectang. DBe [hoc est GB:) est=quadrat. AB, hoc est b^2. $\therefore ax+b^2=x^2$.

Dico 2.) x esse $=\sqrt{\frac{1}{4}a^2+b^2}+\frac{1}{2}a$ Nam Quadr. AC est=$\frac{1}{4}a^2$, & □AB=b^2: quibus per 47.1 æquatur □BC; ergo BC=$\sqrt{\frac{1}{4}a^2+b^2}$. adde CD=CA $=\frac{1}{2}a$ erit BD = BC + CD. Hoc est $x=\sqrt{\frac{1}{4}a^2+b}+\frac{1}{2}a$. *q. e. d.*

Secundus

Secundus CASUS.

$$x^2 = -ax + b^2$$

Solutio $-\frac{1}{2}a$ mult.

$$-\frac{1}{2}a$$

$$+\frac{1}{4}a^2 + b^2$$

$$x = \sqrt{\frac{1}{4}a^2 + b^2} - \frac{1}{2}a = eB$$

Examen seu restitutio.

$$x + \frac{1}{2}a = \sqrt{\frac{1}{4}a^2 + b^2}$$

mult. $x + \frac{1}{2}a$

$$x^2 + ax + \frac{1}{4}a^2 = \frac{1}{4}a^2 + b^2$$

$$\therefore \quad x^2 = -ax + b^2$$

Demonstratio Secundi Casus.

Constructâ figurâ ut ante, sit nunc $eB = x$; unde $DB = a + x$ descriptum autem ab eB Quadratum $eL = x^2$.

Dico nunc 1.) x^2 esse $= b^2 - ax$. Nam $GB = DB = a + x$, & $LB = eB = x$; ergo $GL = De = a$; quæ cum $KL = eB = x$, constituit $\square$ $GK = ax$. Itaque $\square$ G [hoc est $\square$ GK + $\square$ eL] $= x^2 + ax$]. Quod ipsum $\square$ Ge, quum Casu præcedenti ostensum sit $= \square AB = b^2$; erit $x^2 + ax = b^2$ & $x^2 = b^2 - ax$.

Dico 2.) x eſſe $= \sqrt{\frac{1}{4}a^2+b^2}-\frac{1}{2}a$. Nam $Ce = \frac{1}{2}a$, & $eB = x$, unde $CB = \frac{1}{2}a+x = \sqrt{\frac{1}{4}a^2+b^2}$ per præced. ideoque $x = \sqrt{\frac{1}{4}a^2+b^2}-\frac{1}{2}a$. *q. e. d.*

Tertius CASUS.

$x^2 = ax - b^2$

Solutio $\frac{1}{2}a$ mult. $\frac{1}{2}a$

$\frac{1}{4}a^2 - b^2$

$x = \frac{1}{2}a + \sqrt{\frac{1}{4}a^2-b^2}$ Radix major, = BD

& $x = \frac{1}{2}a - \sqrt{\frac{1}{4}a^2-b^2}$ Radix minor, = BE

Habet enim tertius hic Caſus duas veras radices: quarum nunc major, nunc minor, quinetiam ſæpe utraque conſtructioni propoſitæ quæſtionis inſerviet prout uſus docebit.

Examen ſeu reſtitutio.

mult. $x-\frac{1}{2}a = +\sqrt{\frac{1}{4}a^2-b^2}$

$x-\frac{1}{2}a$

$x^2-ax+\frac{1}{4}a^2 = \frac{1}{4}a^2-b^2$

$x^2-ax = -b^2$

$x^2 = ax-b^2$

Demon-

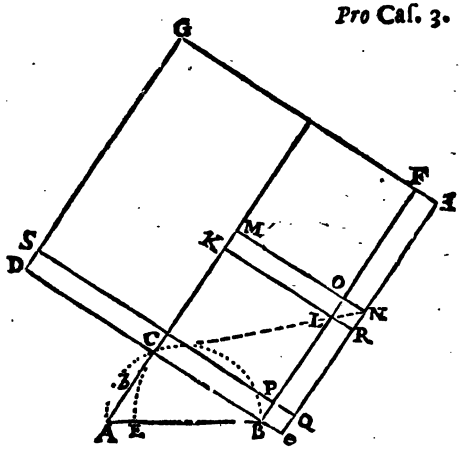
Demonstratio Tertii Casus.

Sit in figurâ sequenti, AB = $\frac{1}{2}a$, & AC=b. Descripto super AB semicirculo, applicetur AC ad peripheriam, jungaturque BC: quâ prolongatâ fiat CD, itemque Ce, = AB.

Sit jam (1.) DB=x, tanquam radix major, super qua constitutum ▫DF erit=x^2. Agatur deinde eH parallela ipsi BF, occurrens GF productæ in H, erit ▭ DH=ax.

Dico nunc 1.) x esse=$\frac{1}{2}a+\sqrt{\frac{1}{4}a^2-b^2}$ per 47.1 & constructionem.

Pro Cas. 3.



Dico 2.) x^2 esse = $ax - b^2$. Constituto enim super Ce = $\frac{1}{2}a$ quadrato CN, erit illud = $\frac{1}{4}a^2$: itemque super CB=$\sqrt{\frac{1}{4}a^2-b^2}$, descripto

pto quadrato CL, erit hoc $=\frac{1}{4}a^2-b^2$. Quare gnomon KNB erit$=b^2$: sed gnomoni huic æquatur $\square$ BH, quod sic ostenditur: Ductâ diametro CN, complementa ML, eL, sunt æqualia per 43. 1 sed ipsi eL est etiam $=\square$ OH: Si enim ex BF=BD auferatur BO $=e$N $=e$C, remanebit OF $=$ BC, $=$ BL: Unde cum & $\square$ OH sit= OK, erit $\square$ BH=gnomoni KNB$=b^2$; ac proinde $\square$DF$=\square$ DH$-\square$ BH; hoc est, $x^2=ax-b^2$. *q. e. d.*

Sit vero (2) Be, $=$AE, $=x$, tanquam radix minor: Super qua constitutum quadratum BQ erit$=x^2$, & $\square$ $eS=ax$: ex quo si auferatur $\square$ BQ, erit reliquum $\square$ BS=BH, $=b^2$. Unde jam reliqua facile patent.

Probl.

Probl. 56. Reperire numerum, quo multiplicato per 6, productoque subtracto ex quadrato ipsius numeri inveniendi, residuum sit 280.

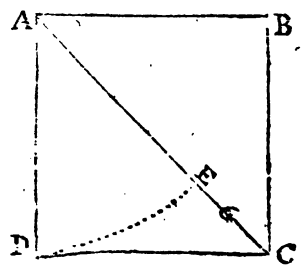
57. Reperire numerum quo multiplicato per 8, productoque addito ad Quadratum ipsius numerei inveniendi, aggregatum sit 660.

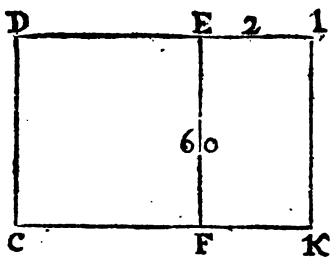
58. Dividere numerum 140 in duas partes, ità ut productum ipsarum partium sit = quadrato ex 56, hoc est 3136.

59. Sint 969 milites in aciem oblongam constituendi, ita ut differentia laterum majoris & minoris sit 40. Quæritur numerus militum cujusque ordinis, in longum & latum.

60. Sint iterum milites 480 in aciem oblongam disponendi, ita ut summa laterum majoris & minoris sit 52. Quæritur numerus militum cujusque ordinis in longum & latum.

61. In Quadrato ABCD datur differentia diametri & lateris, hoc est EC, = 6. Quæritur latus Quadrati.





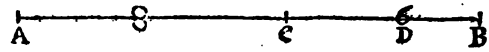
62. Quadrato DF additum eſt [ſub eadem altitudine] rectangulum EK; cujus datur latitudo EL=2, nec non area totius rectanguli compoſiti DK, = 60. Quæritur latus Quadrati.

63. Quidam emit aliquot ulnas panni, pro 70 Florenis; deprehenditque ſi 4 ulnæ amplius fuiſſent, unamquamque ulnam tunc 2 Flor. vilius emtam fuiſſe. Quæritur quot ulnas emerit.

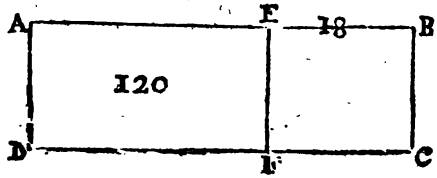
64. Aliquot ſodales debent in diverſorio ſummatim 175 Solidos: poſtquam vero, ante factam ſolutionem, duo ipſorum tacite exceſſere, ſymbola ſingulorum qui remanſerant 10 Solidis adaucta deprehenduntur. Quæritur numerus ſodalium.

65. Dividere numerum 21 in duas partes quarum ſi major dividatur per minorem, & viciſſim minor per majorem; tum prior quotiens multiplicetur per 4, & poſterior per 25, ut numeri producti ſint æquales.

66. Sit



66. Sit linea AB secta in C, ita ut AC sit 8, CB vero 6: oportet eandem lineam AB aliter secare, puta in D, ita ut rectangulum sub AD & DC sit æquale rectangulo sub AC & CB, seu producto ex 8 & 6, hoc est 48. Quæritur intersegmentum CD.



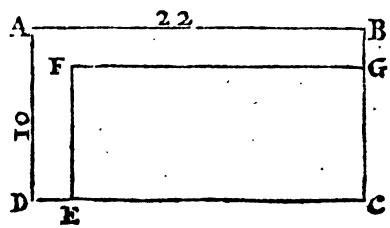
67. Sit hortus rectangularis ABCD, cujus longitudo AB tripla existat latitudinis AD: porro numeratis à B versus A 18 perticis, hoc est BE, ductâque EF parallela ipsi AD, detur area reliqui rectanguli ED 120 perticarum quadratarum. Quæritur dicti horti longitudo AB & latitudo AD.

68. Sint milites 600 in aciem oblongam dispositi: quam dum Colonellus latiorem cupit, deprehendit,

prehendit, si longitudini 10 ordines adimat, latitudini duos ordines accrescere. Quæritur numerus militum per singulos ordines in longum & latum.

69. Aliquis emit equum, quem rursus vendit pro 56 Coronatis, lucraturque tot coronatos ad 100, quot ipsi equus constiterat. Quæritur quanti emtus sit equus.

70. Quidam emit pro 30 Florenis lintea duplicis generis, puta subtilius & crassius: ulnam quidem subtilioris pro tot Florenis quot inde ulnas accepit: tum 28 ulnas crassioris, ita ut 8 ulnæ constent tot Florenis, quot una ulna subtilioris. Quæritur quot ulnas lintei subtilioris, & quo pretio utrumque emerit.



71. In horto quodam rectangulari ABCD, cujus longitudo AB 22, & latitudo AD 10, constituendum sit ambulacrum DG, situ ad latera figuræ parallelo & æquali, ita ut area dicti ambulacri

ambulacri ſeu gnomonis DG æquetur reliquo rectangulo FC [ſive ut gnomon DG ſit dimidia pars totius figuræ ABCD propoſitæ. Quæritur dicti gnomonis latitudo DE, BG.

72. Ex tribus numeris proportionalibus datur medius = 12, & differentia extremorum = 10. Quæruntur ipſi extremi.

73. Ex tribus numeris proportionalibus datur ſumma primi & ſecundi = 10, & differentia ſecundi & tertii = 24. Quæruntur ſinguli.

74. Ex quatuor numeris proportionalibus datur tertius = 12, tum ſumma prima & ſecundi = 8; porro ſi ſecundus numerus ex ſuo Quadrato ſubtrahatur, remanebit quartus. Quæruntur dicti numeri.

75. Ex quatuor numeris continue proportionalibus datur ſumma mediorum = 24, itemque ſumma extremorum = 56. Quæruntur ſinguli [ſuppoſito quod primus numerus ſit minimus reliquorum.]

76. Duæ ruſticæ A & B, ad forum ſimul adferunt 100 ova, vendendoque una tantundem accepit quam altera: aſt A [quæ majora eoque meliora habuerat] dicit ad B, ſi æqualem tecum

ovorum

ovorum numerum habuissem, pro iis 18 Stufros accepissem; cui respondet B, Si paria numero tecum habuissem ova, accepissem pro iis tantummodo 8 Stufros. Quæritur quot ova unaquæque habuerit.

77. Duo rustici, A & B, vendunt frumentum pretio diverso: A quidem 20 Modios; B vero recepit pro uno Modio tot Florenos, quot Modios vendidit: deprehenditque A, si tot Modios vendidisset, quot B accepit Florenos, se tunc acceptorum fuisse 252 Flor. quoniam vero ambo simul receperunt 176 Flor. Quæritur quot Modios A, & quo pretio B, vendiderit.

78. Duo mercatores panni vendunt ulnas 21; primus quidem 1 ulnam tot Coronatis, quanta est $\frac{1}{3}$ ulnarum secundi; secundus autem 1 ulnam tot Coronatis, quanta est $\frac{1}{3}$ ulnarum primi: peractâque venditione connumerant 48 Coronatos. Quæritur quot ulnas singuli & quo pretio vendiderint.

79. Duorum mercatorum serici primus 40 ulnas habet, secundus 90: primus Coronato vendit $\frac{1}{3}$ ulnæ plus quam secundus: tum venditione peractâ ambo numerant Coronatos 42. Quæritur quot ulnas quisque Coronato vendiderit.

80. Reperire

80. Reperire numerum, cujus quadruplo ſi addantur 91, totum ſit ad Quadratum ipſius numeri quæſiti ut 3 ad 4.

81. Reperire numerum à cujus duplo ſi ſubtrahantur 12, Quadratum reliqui minus 1, ſit noncuplum numeri quæſiti.

82. Dividere numerum 19 in duas partes, ita ut ſumma Quadratorum à partibus ſit 193.

83. Dividere 7 in duas partes ita ut differentia Quadratorum quæ fiunt à triplo partis minoris & à duplo partis majoris, ſit 17.

84. Quidam emit volumen lintei, quod rurſus vendendo lucratur 12 Florenos minus $\frac{1}{10}$ expenſi pretii: Deprehenditque ſe hac ratione tandundem lucratum eſſe pro 100, quanti ipſi conſtiterat linteum. Quæritur quanti emtum ſit linteum venditumque.

85. Quidam emit 18 ulnas panni diverſi generis ac coloris, puta rubri & nigri; de unoquoque pro Florenis 40: ſolvitque pro ſingulis ulnis rubri 1 Flor. plus quam pro nigro. Quæritur quot ulnas cujuſque generis emerit.

86. Quidam

86. Quidam emit 120 libras piperis, totidemque zingiberis: ita ut 1 librâ plus zingiberis pro floreno acceperit quam piperis: adeoque totum pretium piperis 6 Florenis superet pretium zingiberis. Quæritur quot libras de unoquoque pro Floreno acceperit.

87. Quidam emit 80 libras piperis, & 36 libr. croci, ita ut pro 8 Flor. acceperit 14 libras piperis plus quam croci pro 26 Flor. estque summa expensi pretii 188 Flor. Quæritur quot libras piperis pro 8 Flor. & quot libras croci pro 26 Florenis acceperit.

88. A & B debent simul 174 florenos; & A quidem singulis diebus persolvit 8 Flor. B vero primo die 1 Flor. secundo 2, tertio 3, & sic consequenter. Quæritur quot diebus totum debitum persolvant; & quantum unusquisque debeat.

89. Quidam peregrinari instituit tot dies quot coronatos habebat; accidit autem ut ipsi singulis sequentibus itineris diebus, totidem ac præcedenti die habuerat, & insuper bini accesserint coronati; absolutoque ut decreverat itinere, deprehendit se omnino 45 coronatos habuisse. Quær. quot initio habuerit.

90. Viator

90. Viator quidam quotidie 9 milliaria conficit, hunc triduo post alter insequitur, qui primo die 4 milliar. conficit, secundo 5, tertio 6, & sic deinceps, singulis diebus 1 milliare lucrando. Quæritur quando hic priorem assequetur.

91. Ex urbibus A & B, quarum distantia 70 milliarium, eodem tempore exit viator, quorum unus singulis diebus conficit milliaria 6; alter vero primo die 2, secundo $2\frac{1}{2}$ tertio 3, & ita deinceps cujusque diei itineri $\frac{1}{2}$ milliare adjiciendo. Quæritur quando sibi mutuo obviabunt, &c.

92. Iterum, ex duabus urbibus A & B, quarum distantia sit 120 milliarium eodem tempore exit viator, quorum primus quotidie conficit 5 milliaria, alter vero quotidie 3 milliar. minus quam est numerus dierum quibus conveniunt. Quæritur ergo quando id fiat.

93. Ex loco A versus B mittitur tabellarius qui singulis diebus 8 milliaria conficit: huic post 27 milliaria confecta alter ex B obviam mittitur, qui quotidie $\frac{1}{20}$ totius viæ seu intervalli locorum A & B conficit, totidemque diebus [quanta nimirum est $\frac{1}{20}$ dictæ distantiæ] completis priori occurrit. Quæritur quantum distent loca A & B.

94. Duo

94. Duo mercatores A & B faciunt ſocietatem, in quam B quidem confert 420 Flor. A vero recipit ex lucro 52 Flor. eſtque ſumma ſortis utriuſque unà cum lucro 854. Flor. Quæritur quantum contulerit A, & quantum ex lucro receperit B.

95. Filius interrogat patrem de ætate ſua; cui pater: Si annis meis demantur 4, reliquum erit triplum annorum tuorum: Si vero ex annis tuis tollatur 1, dimidium reliqui erit radix quadrata annorum meorum. Quær. ætas tam patris quam filii.

96. Reperire duos numeros, quorum ſumma Quadratorum ſit 317, & productum ſi invicem multiplicentur, ſit 154.

97. Reperire duos numeros, quorum productum ſit 108, & differentia Quadratorum 63.

98. Duo ruſtici vendunt frumentum diverſi generis: A quidem 6 Modios; B vero recipit pro ſuo ſummatim 20 Florenos: dicitque B ad A; Ecce ſi addamus numeros meorum Modiorum & tuorum Florenorum, ſumma erit 28: reſpondet A, Et ſi addam Quadratum meorum Florenorum Quadrato tuorum Modiorum, aggregatum erit 424. Quær. quot Modios vendiderit B, & quot Florenos receperit.

99. Re-

99. Reperire duos numeros, quorum primus +2, multiplicatus in secundum —3, producat 110: & vicissim primus —3, multiplicatus in secundum +2, producat 80.

Tabulæ Poteſtatum Numericarum

√	Qua	√	Qua.	√	Quad.	√	Quad.	√	Quadr.
1	1	21	441	41	1681	61	3721	81	6561
2	4	22	484	42	1764	62	3844	82	6724
3	9	23	529	43	1849	63	3969	83	6889
4	16	24	576	44	1936	64	4096	84	7056
5	25	25	625	45	2025	65	4225	85	7225
6	36	26	676	46	2116	66	4356	86	7396
7	49	27	729	47	2209	67	4489	87	7569
8	64	28	784	48	2304	68	4624	88	7744
9	81	29	841	49	2401	69	4761	89	7921
10	100	30	900	50	2500	70	4900	90	8100
11	121	31	961	51	2601	71	5041	91	82281
12	144	32	1 04	52	2704	72	5184	92	8464
13	169	33	1089	53	2809	73	5329	93	8649
14	196	34	1156	54	2916	74	5476	94	8836
15	225	35	1225	55	3025	75	5625	95	9025
16	256	36	1296	56	3136	76	5776	96	9216
17	289	37	1369	57	3249	77	5929	97	9409
18	324	38	1444	58	3364	78	6084	98	5604
19	361	39	1521	59	3481	79	6241	99	9801
20	400	40	1600	60	3600	80	6400	100	10000
√c.	Cub.	√c.	Cub.	√c.	Cub.	v√	Q.qu.	√	Q.qua.
1	1	11	1331	21	9261	1	1	11	14641
2	8	12	1728	22	10648	2	16	12	20736
3	27	13	2197	23	12167	3	81	13	28561
4	64	14	2744	24	13824	4	256	14	38416
5	125	15	3357	25	15625	5	625	15	50625
6	216	16	4096	26	17576	6	1296	16	65536
7	343	17	4913	27	19683	7	2401	17	83521
8	512	18	5832	28	21952	8	4096	18	104976
9	729	19	6859	29	24389	9	6561	19	130321
10	1000	20	8000	30	27000	10	10000	20	160000

Tab. reductioræ Numerorum irrationalium.

$\sqrt{6} = 2\sqrt{1\frac{1}{2}}$	$\sqrt{75} = 5\sqrt{3}$	$\sqrt{200} = 5\sqrt{8}$	$\sqrt{486} = 9\sqrt{6}$
$\sqrt{8} \quad 2\sqrt{2^2}$	$\sqrt{80} \quad 4\sqrt{5}$	$\sqrt{216} \quad 6\sqrt{6}$	$\sqrt{490} \quad 7\sqrt{10}$
$\sqrt{10} \quad 2\sqrt{2\frac{1}{2}}$	$\sqrt{90} \quad 3\sqrt{10}$	$\sqrt{243} \quad 9\sqrt{3}$	$\sqrt{500} \quad 10\sqrt{5}$
$\sqrt{12} \quad 2\sqrt{3^2}$	$\sqrt{96} \quad 4\sqrt{6}$	$\sqrt{245} \quad 7\sqrt{5}$	$\sqrt{512} \quad 8\sqrt{8}$
$\sqrt{18} \quad 3\sqrt{2}$	$\sqrt{98} \quad 7\sqrt{2}$	$\sqrt{250} \quad 5\sqrt{10}$	$\sqrt{539} \quad 7\sqrt{11}$
$\sqrt{20} \quad 2\sqrt{5}$	$\sqrt{99} \quad 3\sqrt{11}$	$\sqrt{252} \quad 6\sqrt{7}$	$\sqrt{567} \quad 9\sqrt{7}$
$\sqrt{24} \quad 2\sqrt{6}$	$\sqrt{108} \quad 6\sqrt{3}$	$\sqrt{275} \quad 5\sqrt{11}$	$\sqrt{600} \quad 10\sqrt{6}$
$\sqrt{27} \quad 3\sqrt{3}$	$\sqrt{112} \quad 4\sqrt{7}$	$\sqrt{288} \quad 6\sqrt{8}$	$\sqrt{640} \quad 8\sqrt{10}$
$\sqrt{28} \quad 2\sqrt{7}$	$\sqrt{125} \quad 5\sqrt{5}$	$\sqrt{294} \quad 7\sqrt{6}$	$\sqrt{648} \quad 9\sqrt{8}$
$\sqrt{32} = 4\sqrt{2}$	$\sqrt{128} = 8\sqrt{2}$	$\sqrt{300} = 10\sqrt{3}$	$\sqrt{700} = 10\sqrt{7}$
$\sqrt{40} \quad 2\sqrt{10}$	$\sqrt{147} \quad 7\sqrt{3}$	$\sqrt{320} \quad 8\sqrt{5}$	$\sqrt{704} \quad 8\sqrt{11}$
$\sqrt{44} \quad 2\sqrt{11}$	$\sqrt{150} \quad 5\sqrt{6}$	$\sqrt{343} \quad 7\sqrt{7}$	$\sqrt{800} \quad 10\sqrt{8}$
$\sqrt{45} \quad 3\sqrt{5}$	$\sqrt{160} \quad 4\sqrt{10}$	$\sqrt{360} \quad 6\sqrt{10}$	$\sqrt{810} \quad 9\sqrt{10}$
$\sqrt{48} \quad 4\sqrt{3}$	$\sqrt{162} \quad 9\sqrt{2}$	$\sqrt{384} \quad 8\sqrt{6}$	$\sqrt{891} \quad 9\sqrt{11}$
$\sqrt{50} \quad 5\sqrt{2}$	$\sqrt{175} \quad 5\sqrt{7}$	$\sqrt{392} \quad 7\sqrt{8}$	$\sqrt{1000} \quad 10\sqrt{10}$
$\sqrt{54} \quad 3\sqrt{6}$	$\sqrt{176} \quad 4\sqrt{11}$	$\sqrt{396} \quad 6\sqrt{11}$	$\sqrt{1100} \quad 10\sqrt{11}$
$\sqrt{63} \quad 3\sqrt{7}$	$\sqrt{180} \quad 6\sqrt{5}$	$\sqrt{405} \quad 9\sqrt{5}$	$\sqrt{1200} \quad 10\sqrt{12}$
$\sqrt{72} \quad 3\sqrt{8}$	$\sqrt{192} \quad 8\sqrt{3}$	$\sqrt{448} \quad 8\sqrt{7}$	
$2\sqrt{2} = \sqrt{8}$	$2\sqrt{5} = \sqrt{20}$	$2\sqrt{7} = \sqrt{28}$	$2\sqrt{10} = \sqrt{40}$
$3\sqrt{2} \quad \sqrt{18}$	$3\sqrt{5} \quad \sqrt{45}$	$3\sqrt{7} \quad \sqrt{63}$	$3\sqrt{10} \quad \sqrt{90}$
$4\sqrt{2} \quad \sqrt{32}$	$4\sqrt{5} \quad \sqrt{80}$	$4\sqrt{7} \quad \sqrt{112}$	$4\sqrt{10} \quad \sqrt{160}$
$5\sqrt{2} \quad \sqrt{50}$	$5\sqrt{5} \quad \sqrt{125}$	$5\sqrt{7} \quad \sqrt{175}$	$5\sqrt{10} \quad \sqrt{250}$
$6\sqrt{2} \quad \sqrt{72}$	$6\sqrt{5} \quad \sqrt{180}$	$6\sqrt{7} \quad \sqrt{252}$	$6\sqrt{10} \quad \sqrt{360}$
$7\sqrt{2} \quad \sqrt{90}$	$7\sqrt{5} \quad \sqrt{245}$	$7\sqrt{7} \quad \sqrt{343}$	$7\sqrt{10} \quad \sqrt{490}$
$8\sqrt{2} \quad \sqrt{128}$	$8\sqrt{5} \quad \sqrt{320}$	$8\sqrt{7} \quad \sqrt{448}$	$8\sqrt{10} \quad \sqrt{640}$
$9\sqrt{2} \quad \sqrt{162}$	$9\sqrt{5} \quad \sqrt{405}$	$9\sqrt{7} \quad \sqrt{567}$	$9\sqrt{10} \quad \sqrt{810}$
$10\sqrt{2} \quad \sqrt{200}$	$10\sqrt{5} \quad \sqrt{500}$	$10\sqrt{7} \quad \sqrt{700}$	$10\sqrt{10} \quad \sqrt{1000}$
$2\sqrt{3} = \sqrt{12}$	$2\sqrt{6} = \sqrt{24}$	$2\sqrt{8} = \sqrt{32}$	$2\sqrt{11} = \sqrt{44}$
$3\sqrt{3} \quad \sqrt{27}$	$3\sqrt{6} \quad \sqrt{54}$	$3\sqrt{8} \quad \sqrt{72}$	$3\sqrt{11} \quad \sqrt{99}$
$4\sqrt{3} \quad \sqrt{48}$	$4\sqrt{6} \quad \sqrt{96}$	$4\sqrt{8} \quad \sqrt{128}$	$4\sqrt{11} \quad \sqrt{176}$
$5\sqrt{3} \quad \sqrt{75}$	$5\sqrt{6} \quad \sqrt{150}$	$5\sqrt{8} \quad \sqrt{200}$	$5\sqrt{11} \quad \sqrt{275}$
$6\sqrt{3} \quad \sqrt{108}$	$6\sqrt{6} \quad \sqrt{216}$	$6\sqrt{8} \quad \sqrt{288}$	$6\sqrt{11} \quad \sqrt{396}$
$7\sqrt{3} \quad \sqrt{147}$	$7\sqrt{6} \quad \sqrt{294}$	$7\sqrt{8} \quad \sqrt{392}$	$7\sqrt{11} \quad \sqrt{539}$
$8\sqrt{3} \quad \sqrt{192}$	$8\sqrt{6} \quad \sqrt{384}$	$8\sqrt{8} \quad \sqrt{512}$	$8\sqrt{11} \quad \sqrt{704}$
$9\sqrt{3} \quad \sqrt{243}$	$9\sqrt{6} \quad \sqrt{486}$	$9\sqrt{8} \quad \sqrt{648}$	$9\sqrt{11} \quad \sqrt{891}$
$10\sqrt{3} \quad \sqrt{300}$	$10\sqrt{6} \quad \sqrt{600}$	$10\sqrt{8} \quad \sqrt{800}$	$10\sqrt{11} \quad \sqrt{1000}$
$2\sqrt{c.2} = \sqrt{c.16}$	$2\sqrt{c.3} = \sqrt{c.34}$	$2\sqrt{c.4} = \sqrt{c.32}$	$2\sqrt{c.5} = \sqrt{c.40}$
$3\sqrt{c.2} \quad 54$	$3\sqrt{c.3} \quad 81$	$3\sqrt{c.4} \quad 108$	$3\sqrt{c.5} \quad 135$
$4\sqrt{c.2} \quad 128$	$4\sqrt{c.3} \quad 192$	$4\sqrt{c.4} \quad 256$	$4\sqrt{c.5} \quad 220$
$5\sqrt{c.2} \quad 250$	$5\sqrt{c.3} \quad 375$	$5\sqrt{c.4} \quad 500$	$5\sqrt{c.5} \quad 625$
$6\sqrt{c.2} \quad 432$	$6\sqrt{c.3} \quad 648$	$6\sqrt{c.4} \quad 864$	$6\sqrt{c.5} \quad 1080$
$7\sqrt{c.2} \quad 686$	$7\sqrt{c.3} \quad 1029$	$7\sqrt{c.4} \quad 1372$	$7\sqrt{c.5} \quad 1715$
$8\sqrt{c.2} \quad 1024$	$8\sqrt{c.3} \quad 1536$	$8\sqrt{c.4} \quad 2048$	$8\sqrt{c.5} \quad 2560$
$9\sqrt{c.2} \quad 1458$	$9\sqrt{c.3} \quad 2187$	$9\sqrt{c.4} \quad 2910$	$9\sqrt{c.5} \quad 3645$
$10\sqrt{c.2} \quad 2000$	$10\sqrt{c.3} \quad 3000$	$10\sqrt{c.4} \quad 4000$	$10\sqrt{c.5} \quad 5000$

LOGISTICA LINEARIS

Pro Conſtructione

Aquationum Geometrica.

PROPOSITIO I.

Lineæ rectæ datæ datam rectam addere earumque exhibere ſummam.

SINT addendæ a & b; ſumma erit $a+b=d$.

PROP. II.

Ex data recta majore minorem ſubtrahere earumque exhibere differentiam.

SUbtrahatur c ex b; differentia erit $b-c=f$.

PROP.

Prop. III.

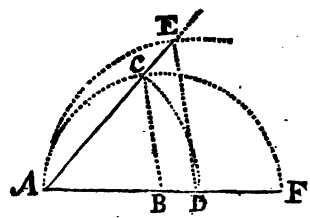
Datis duabus rectis tertiam proportionalem invenire [idq; quàm fieri poſſit ſumma exactitudine.]

Caſus 1.

Si datarum differentia ſit mediocris, ſeu non nimis vaſta.

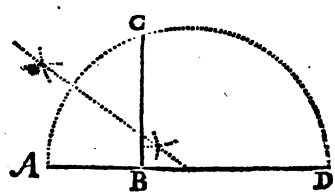
Intervallo primæ AB deſcribatur ex B ſemicirculus ACF, in quo ex A accommodetur ſecunda AC, eaque ultra C infinite producatur: tum eadem ſecunda ex A collocetur in Diametrum ſemicirculi, puta in D, atque ex D, intervallo DA deſcriptâ peripherâ, in productâ AC dabit punctum E, eritque AE tertia proportionalis quæſita.

AB $= a$, prima.
AC
AD $= b$, ſecunda.
AE $= \frac{b^2}{a}$, tertia quæſ.



Vel etiam ita.

AB, prima.
BC, ſecunda.
BD, tertia quæſit.

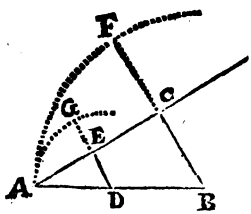


Quin circa primum hunc Caſum, ubi ſcil. datarum differentia non eſt juſto major, variæ operationes mediæque commode adhiberi poſſunt [uti & in ſubſeq. *Prop.* IV.] at non pariter in cæteris Caſibus.

Caſus 2.

Si datarum differentia ſit nimis magna; & prima major exiſtat quam altera.

Intervallo primæ AB, & B fiat angulus 60 grad. ABF; e cujus altero crure BF abſcindatur ſecunda BC, ducaturque AC. Fiat nunc AD= ſecundæ BC, atque ex D, intervallo DA [vel quolibet alio] deſcribatur angulus itidem 60 grad. ADG, cujus alterum crus DG à recta AC ſecabitur in E, eritque DE tertia proportionalis quæſita.



AB = *a* prima.
BC
AD = *b* ſecunda.
DE = $\frac{b^2}{a}$ tertia quæſita.

Caſus

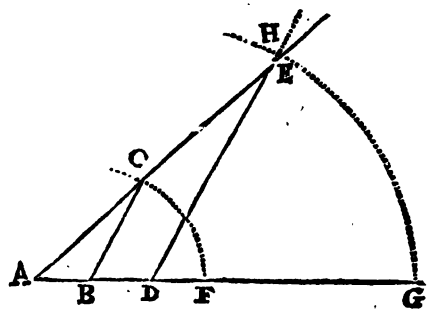
Caſus 3.

Si datarum differentia ſit nimis magna, & prima minor exiſtat quam altera.

1. Ex B, termino primæ, intervallo ſecundæ BC, deſignetur angulus 60 grad. FBC, & ex A per C agatur recta infinita. 2. Ex D termino ſecundæ, intervallo ſatis magno deſignetur iterum angulus 60 grad. GDA, ductâque rectâ uſque dum occurrat infinitæ AC in E, erit DE tertia proportionalis quæſita.

Cætera ad hanc Prop. obſervanda vid. ad ſubſeq.

$AB = a$, pri.
BC
$AD = b$, ſec.
$DE = \frac{b^2}{a}$, ter.
quæſita.



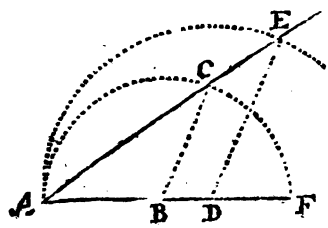
Prop. IV.

Datis tribus rectis quartam proportionalem invenire.

Caſus 1.

Si datarum [puta primæ & alterutrius reliquarum] differentia non ſit immodica, & præſertim ſi prima minor exiſtat altera reliquarum.

Intervallo primæ AB, ex B deſcribatur ſemicirculus ACF, in quo ex A accommodetur ſecunda AC, eaque ulterius infinite producatur: tum tertia ex A collocetur in Diametrum ſemicirculi, puta in D; atque ex D, intervallo DA, deſcripta peripheria ſignabit in producta AC punctum E, eritque AE quarta proportionalis quæſita.



AB = *a* prima
AC = *b* ſecunda
AD = *c* tertia
AE = $\frac{bc}{c}$ quarta quæſita.

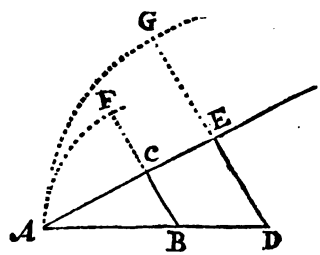
Caſus

Caſus 2.

Si datarum [primæ & utriuſlibet aut alterutrius reliquarum] differentia ſit immodica; & prima major exiſtat alterutra vel utravis reliquarum.

Intervallo primæ AB, ex B fiat angulus 60 grad. ABF, e cujus altero crure BF abſcindatur ſecunda BC, ducaturque ex A per C recta infinita: Collocetur jam tertia ex A in D; & ex D, intervallo DA [vel quolibet alio] fiat itidem angulus 60 grad. ADG, cujus alterum crus DG ſecabit productam AC, eritque DE quarta proportionalis quæſita.

$AB = a$, prima
$BC = b$, ſecunda
$AD = c$, tertia
$DE = \frac{bc}{a}$ quarta quæſita.



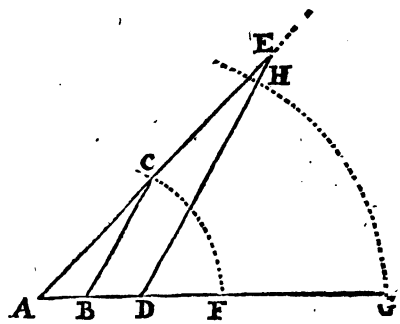
[*Nota*, Si prima non ſit major utràque reliquarum, ſed duntaxat alterutrâ, uti hoc loco, operatio commode etiam inſtituetur juxta *Caſ.* 1. ſecundâ nimirum & tertiâ invicem perimutatis.]

Caſus

Caſus 3.

Si datarum [primæ & utriuſlibet reliquarum] *differentia ſit immodica; & prima exiſtat minor.*

1. Ex B, termino primæ, intervallo ſecundæ BC, deſignetur angulus 60 grad. FBC; & ex A per C agatur recta infinita. 2. Ex D, termino tertiæ, intervallo ſatis magno deſignetur iterum angulus 60 grad. GDH, ductâque recta DH, uſque dum occurrat infinitæ AC in E, erit DE quarta proportionalis quæſita:



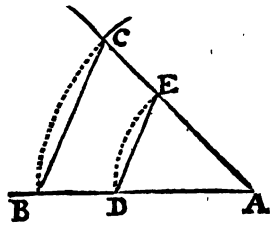
AB $=a$, prim.
BC $=b$, ſec.
AD $=c$, tertia
DE $=\frac{bc}{a}$ quartus quæſ.

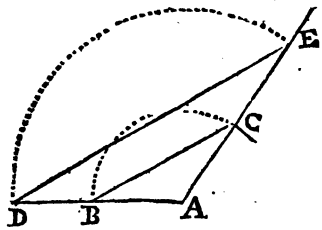
Vel, aſſumatur dupla primæ, & prodibit dimidia quæſitæ: ac tunc ſæpe operatio ad *Caſum* 1. reduci poterit.

§. Cæterum niſi prima datarum ſit juſto brevior, praxis generatim [hujuſce & præced. propoſ.] non minus accurata quam expedita eſt quæ ſit per triangulum æquicrurum uti vulgò notum.

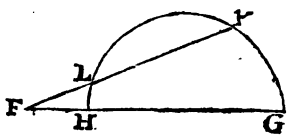
AB Prima

AB prima
BC ſecunda
AD tertia
DE quarta quæſ.

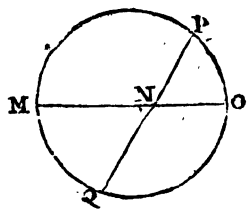




§. Interim notari quoque poſſunt modi ſequentes ex 35. & 36. III. *Euclid.*



FK prima
FG ſecunda
FH tertia
FL quarta



NQ prima
MN ſecunda
NO tertia
NP quarta quæſita.

§. *Obſervandum*

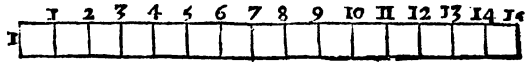
§. *Obſervandum* tamen, non quemlibet modorum recenſitorum cullibet Problemati conſtruendo, quantum ad demonſtrationem concinnandam attinet, æquè commodè aut rectè adhiberi, ſed plerumque Problemata ſuam quodque conſtructionem requirere aut ſecum ferre; at nobis hic ſaltem conſtructionis exactitudo curæ eſſe debuit.

PROP. V. [Lemma.]

Lineam rectem datam in rectam alteram ducere ſeu multiplicare, ut productum ſit quaſi linea. Seu Geometricè loquendo.

Super data recta rectangulum conſtituere; quod [valore ſuo numerico] *æquipolleat ipſi linea propoſitæ.*

Fiet hoc ſi recta multiplicans ſit unitas, ſeu ſi pro altero latere conſtituendi plani aſſumatur recta quæ unitatem referat, uti hic videre eſt.



§. Pari ratione conſtituetur quantitas quolibet dimenſionum, quæ tamen unicam duntaxat dimenſionem repræſentet.

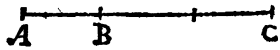
Sic etiam quælibet quantitas in lineam multiplicabitur; ut productum æquipolleat quantitati datæ.

PROP.

Prop. VI.

Datam rectam per numerum multiplicare.

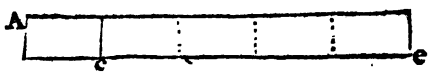
UT ſi data recta A*b* multiplicanda ſit per 3 prodibit recta AC.



Prop. VII.

Datum planum per numerum multiplicare.

UT ſi datum planum A*c* multiplicandum ſit per 5, prodibit planum *Ae*.



Prop. VIII.

Datum planum per lineam dividere.

[Quod eſt fractionem in proportionum reſolvere.]

SIT ab ▭ dividendum per *c*, fiet, ut *c* diviſor ad *a*: Ita *b* ad $\frac{ab}{c}$ ſeu quotum quæſit. $=x$.

Nota:

Nota: Si planum dividendum sit compositum, ut $\frac{a^2+b^2}{c}$, $\frac{c^2+b^2-ab}{2a}$, &c.

Prop. IX.

Datam lineam rectam per rectam dividere.

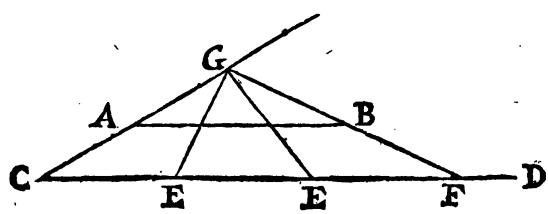
COgitetur super data recta constitutum esse planum, cujus alterum latus sit unitas [per V. præced.] atque tunc fiat juxta VIII. præced.

Ut si dividere oporteat b per a; fiet ut a ad b; ita unitas $= e$ ad $\frac{eb}{a} = \frac{b}{a}$.

Prop. X.

Datam rectam per numerum dividere.

UT si dividenda sit recta AB per 3, sive pars tertia ipsius exhibenda. Ducatur CD parallela datæ AB; dein quovis intervallo fiant CE, EE, & EF æquales inter se & numero dato. Demum jungantur CA & FB donec coeant ut ad G. Postremo junge GE, EE, &c. & habebis lineam divisam prout oportet.

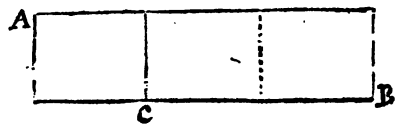


PROP.

PROP. XI.

Datum planum per numerum dividere.

SIT propoſitum planam AB dividendum per 3, Quotus erit tertia pars plani propoſiti, puta AC,



PROP. XII.

Datum planum in aliud tranſmutare quod habeat datum latus; ſeu *datum planum ad datam longitudinem vel latitudinem reducere.*

SIT ▭ *bc* tranſmutandum in aliud, cujus unum latus ſit = datæ rectæ *d*; ſeu ▭ *bc* ſit reducendum ad longitudinem *d*, quæritur latitudo.

Dividatur [per VIII. præced.] propoſitum ▭ *bc* per datum latus *d*, & prodibit alterum latus quæſitum; hoc eſt, fiat ut *d* ad *b*, ita *c* ad quartam = *f*, = lateri quæſito, unde jam *df* = *bc*.

§. Patet hinc, qua ratione quælibet quantitas quotcunque dimenſionum poſſit reduci ad datas literas, ſeu ad datam laterum [per unum] magnitudinem.

Sit

Sit quantitas acb^2 ita tranſmutanda, ut in ea ter reperiatur litera a [ſeu a cubicum]. Aſſumpto itaque primum quadrato b^2, fiet ut a ad b, ita b ad tertiam $= d$; unde ad $= b^2$, & $acad$ ſeu $aacd = acb^2$. Tum porro aſſumpto quadrato cd fiet ut a ad c, ita d ad quartam $= f$; unde $af = cd$, & $aaaf$ ſeu $a^3f = acb^2$, quod deſiderabatur.

Prop. XIII.

Datum planum in lineam exponere: ſeu *datum planum in aliud convertere quod habeat rationem lineæ* [vel ut vulgo effertur] *Lineam in lineam multiplicare juxta datam unitatem.*

Dividatur propoſitum planum per unitatem, Quotus dabit quæſitum. Sit Quadrat. ab exponendum in lineam, fiet, ut unitas, $= e$, ad a, ita b ad quartam $= f$; unde $ef = ab$, & ef quoque $=$ ipſi lineæ f, per V. præced.

§. Patet hinc qua ratione quælibet quotcunque dimenſionum quantitas in lineam exponi, ſeu ad valorem linearem redigi poſſit. Sit ex. gr. quantitas a^3b^2 exponenda in lineam.

Suppoſitâ unitate $= e$, aſſumptoque primum Quadrato a^2, fiet; ut e ad a, ità a ad tertiam $= d$, unde $ed = a^2$. Similiter aſſumpto Quadrato b^2, fiet; ut e ad b, ità b ad tertiam $= f$, unde $ef = b^2$, & $eedf = a^3b^2$: itaque tandem aſſumpto Rectangulo df fiet: ut e ad d, ità f ad quartam $= g$, unde $eg = df$; & conſequentur $e^2g = a^3b^2$; & ſimul

mul etiam e^2g = ipſi g numericè; quod expetebatur.

§. Quod ſi vero in propoſito exemplo [a^2b^2] ipſa quantitas a ſumatur pro unitate, brevius ad ſcopium venietur; nimirum reducendo duntaxat Quadrat. b^2 ut ſequitur: ut a ad b, ita b ad tertiam $= k$; unde $ak = b^2$; ac proinde $a^2k = a^2b^2$ = ipſi k.

PROP. XIV.

Datum quantitatem quancunque per quamlibet alium dividere. [ſeu *Fractionem quamcunque in proportionem reſolvere.*]

Caſus 1.

Cum Diviſor pauciores dimenſiones habet quam dividendus. Quæ eſt Diviſio ordinaria, ut $\frac{bd^2}{ac}$, $\frac{a^2df}{bg}$, $\frac{a^2b^2+bf^3}{abf-af^2}$.

Caſus 2.

Cum Diviſor & dividendus dimenſionibus pares exiſtunt, ut $\frac{ab}{c^2}$, $\frac{d^2f}{ab^2}$, $\frac{b^2ac}{f^2d^2}$.

Tunc in dividendo adſciſcenda eſt unitas per V. præced. quæ ſit ex. gr. $= e$; erit igitur $\frac{eab}{c^2} = \frac{ab}{c^2}$ propoſito; & ſic etiam in cæteris.

Caſus 3.

Cum Diviſor plures dimenſiones habet quam dividendus, ut $\frac{b}{a^2}$, $\frac{f^2}{cd^2}$, $\frac{ac^2}{b^2f}$.

Tunc in dividendo adſciſcenda eſt unitas quoties opus, puta uſque dum ipſe dividendus numero dimenſionum ſuperet diviſorem. Suppoſita igitur unitate $= e$ ut ante, ponetur.

$$\frac{e^2b}{a^2} = \frac{b}{a^2}, \quad \frac{e^2f^2}{cd^2} = \frac{f^2}{cd^2}, \text{ &c.}$$

Caſus 4.

Cum Diviſor pauciores dimenſiones habet quam ratio ſeu ſpecies quoti exigit.

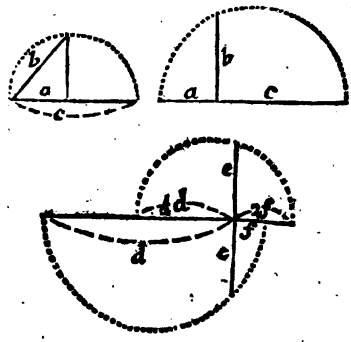
Ut ſi propoſitum ſit $\frac{ab^2}{c}$, & quotus debeat eſſe linea. Tunc in diviſore adſciſcenda eſt unitas, ut ſit $\frac{ab^2}{ec} = \frac{ab^2}{e}$, &c.

Prop. XV.

Datis duabus rectis mediam proportionalem invenire.

DEntur a & c; prodibit media $= b$, per conſtructiones vulgo notas,

Quodſi



Quodſi alterutra datarum ſit nimis parva, ſumatur ejuſdem dupla vel tripla, & alterius dimidia vel tertia pars, proveniet enim ipſa quæſita, ut ante.

Dentur exempgr. *d* & *f*, media erit *e*, utroque modo reperta, ut figura oſtendit.

Prop. XVI.

Dati plani [ſolitarii] *latus quadratum reperire.*

Media proportionalis inter propoſiti plani [puta rectanguli] longitudinem & latitudinem exhibet latus quadratum quæſitum.

Prop. XVII.

Dati plani plano aucti vel diminuti latus quadratum invenire.

Ut ſi dentur a^2+b^2, a^2-b^2, $a^2+b^2-c^2$, &c. Conſtructio ex 47. I. & 31. III. patet.

PROP. XVIII.

Dati plani per numerum multiplicati vel diviſi latus quadratum reperire.

UT ſi dentur $3a^2$, $\frac{1}{2}b^2$, $\frac{1}{3}ab$, &c.
Conſtructio plerumque duplici ratione perfici poteſt per 47. I. & 13. VI.

PROP. XIX.

Dati plani tam plano quam numero affecti [juxta præced. XVII, XVIII.] *latus quadratum invenire.*

UT ſi dentur $2a^3+\frac{1}{3}b^2$, $5b^2+d^2-\frac{1}{2}c^2$, &c.
Conſtructio patet ex præcedentibus.

PROP. XX.

Datæ quantitatis cujuſcunque, ad duas dimenſiones diviſæ latus quadratum invenire.

UT ſi detur $\frac{ab^2}{c}$ $\frac{a^2d^2}{bc}$ $\frac{c^2f^2+d^4}{c^2-d^2}$ &c. Fiat b ad q, c ad a: deinde medium proportionale inter b & q erit latus quæſitum. Et ſic de cæteris.

PROP.

PROP. XXI.

Ex data linea recta radicem quadratam extrahere.

INtelligatur ſuper data recta conſtitutum eſſe planum cujus alterum latus ſit unitas [quæ quidem interdum vera ſeu determinata requiritur; interdum autem quævis hypothetica ac pro lubitu aſſumpta eidem quæſito ſatisfaciet] ac tum [juxta XV. & XVI] media proportionalis inter dictam unitatem & rectam propoſitam dabit radicem quæſitam.

Quod ſit data unitas ſi nimis parva, aſſumetur ejus duplum vel triplum, & è contra propoſitæ lineæ dimidium vel triens, prout ad Propoſ. XV. monitum fuit.

PROP. XXII.

Ex data quantitate trium dimenſionum radicem quadratam extrahere.

1. SIT quærenda $\sqrt{a^3}$, quæ cum nihil aliud ſit quam quadrat. ex $\sqrt{a^2}$ [hoc eſt a] & $\sqrt{a}$, ſeu $\sqrt{a^1}$; idem eſt ac $a\sqrt{a}$; ideo primum quæratur $\sqrt{a}$ per præced. XXI. quæ in a ducta planum conſtituet; quo per XIII. ad rationem lineæ redacto habebitur quæſitum.

Vel data quantitas a^3 primum exponatur in lineam, ac tunc fiat extractio juxta XXI.

Notandum autem, pro hujusmodi extractione (uti & in reliquis Casibus hujusce Propositionis) unitatem non posse assumi pro lubitu, sed ab ipsa quantitate a dependere.

2. Sit quærenda $\sqrt{ab^2}$, quæ est = quadrato ex b & $\sqrt{a}$, quo in lineam exposito habebitur quæsitum.

3. Sit quærenda $\sqrt{bcd}$, quæ est=rectangulo ex $\sqrt{b}$ & $\sqrt{cd}$. Itaque per XVI. quæritur $\sqrt{cd}$, & per XXI. $\sqrt{b}$; indeque constitutum quadratum exponatur in lineam per XIII. quæ ipsa erit $\sqrt{}$ quæsita.

§. Pro declaratione per numeros esto $a=4$; $b=3$ $c=2$; $d=5$.

Prop. XXIII.

Ex data quantitate quatuor dimensionum radicem quadratam extrahere.

Nota, HOC loco idem unitas determinatur per datas quantitates, nec pro lubitu assumi potest.

1. Sit quærenda $\sqrt{a^4}$, quæ est=a^2; ideoque a^2 per XII. exponatur in lineam & habebitur quæsitum.

2. Sit quærenda $\sqrt{a^2b^2}$, quæ est=ab, ideoque ut prius.

3. Sit

3. Sit quærenda $\sqrt{b^2dc}$, quæ eſt$=b\sqrt{dc}$, quæratur $\sqrt{dc}$ per XVI. quæ cum b conſtituet planum, quod expoſitum in lineam per XIII. dabit radicem quæſitam.

4. Sit quærenda $\sqrt{abcd}$; quæratur $\sqrt{ab}$, itemque $\sqrt{cd}$ per XVI. & conſtitutum ab utraque planum redigatur in lineam per XIII. habebiturque radix quæſita.

PROP. XXIV.

Ex data quantitate quatuor dimenſionum radicem biquadraticam extrahere.

1. *Modus qui fit ope unitatis.*

QUÆ hoc loco pro lubitu major minorve aſſumi poteſt, ſemper enim ultimo vera radix prodibit; ac conſequenter etiam litera aliqua ipſius datæ quantitatis pro unitate accipi poterit, quod operationi non parum compendii adferet.

Regula generalis pro omnibus Caſibus.

Quæratur primum per præced. XXIII. $\surd$ quadr. propoſitæ quantitatis; ac tum ex iſta radice iterum radix extrahatur per XXI. quæ erit $\surd$ quæſita.

2. Modus abſque ope unitatis.

1. Pro $\sqrt{a^4}$ nulla opus operatione, eſt enim ea $=a$.

2. Pro $\sqrt{a^2b^2}=\sqrt{ab}$ media proportionalis inter a & b dabit quæſitum.

3. Pro $\sqrt{b^2dc}=\sqrt{b}\sqrt{\sqrt{dc}}$ media proportionalis inter b & $\sqrt{dc}$ dabit quæſitum.

4. Pro $\sqrt{abcd}\sqrt{\sqrt{ab}\sqrt{cd}}$ media proportionalis inter $\sqrt{ab}$ & $\sqrt{cd}$ dabit quæſitum.

PROP. XXV.

Ex data quantitate quatuor dimenſionum compoſita vel diminuta radicem biquadraticam extrahere.

HIC itidem ut in præced. unitas quæcunque ad libitum aſſumi poteſt; commodiſſime tamen illa litera quantitatis propoſitæ, quæ in ea plurimum occurrit, unitatis vice fungetur

1. Pro $\sqrt{b^2a^2+b^4}$. Aſſumpto b pro unitate exponatur per eam a^2 in lineam [per XIII.] quæ vocetur f. Exponatur item b^2 in lineam, quæ erit $=b$; unde $bf=b^2a^2$, & $\sqrt{bf}=\sqrt{b^2a^2}$; ſed & $\sqrt{bf}=a$; cui additâ per XVII. præced. radice $b^4=b^2=b$ (quum b ſupponatur $=1$) habebitur primum $\sqrt{b^2a^2+b^4}$. Ex quæ ſi rurſum [per XXI.] radix

radix extrahatur habebitur $w\overline{b^2a^2+b^4}$ quæſita.

Hinc conſtructionis compendium: Ex $\sqrt{a^2+b^2}$ extracta radix ope $b=1$, dabit w quæſitam.

2. Pro $w\overline{a^4+b^4}$. Poſita $a=1$.

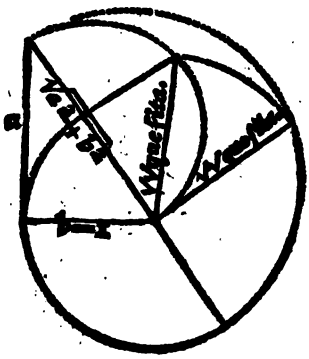
3. Pro $w\overline{bcd^2+bdc^2}$. Poſito $d=1$, &c.

Regula generalis hujuſce Propoſitionis.

Exponantur plana propoſitarum quantitatum in lineas, per XIII. cum quibus deinde porro proœedatur juxta XVI, XVII. Pro radice nimirum quadrata; ac denique juxta XXI. pro rad. Quadr.

Sequuntur Conſtructiones.

§. Pro $w\overline{b^2a^2+b^4}$. Poſito $b=1$.

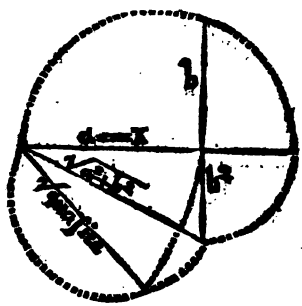


Porro idem obtinebitur quidem ſi a ponatur$=1$, ſed operatione minus compendioſa.

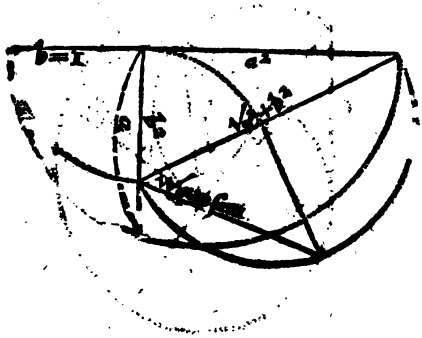
§. Pro

§. Pro $w\sqrt{a^4+b^4}$. 1. Posito $a=1$.

Erit $a^2=a$, jam pro b^2, ut $a=1$ ad b, ita b ad $\frac{b^2}{1}$ unde requiritur $\sqrt{a^2+b^2}$; ex qua denuo extracta $\sqrt{}$ ope $a=1$, dabit quæsitam.



2. Posito $b=1$, unde $b^2=b$, &c.



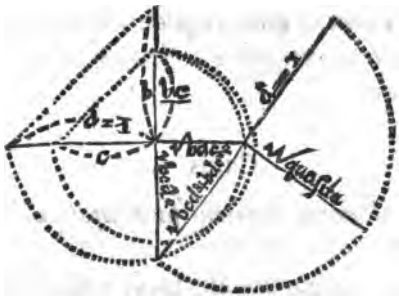
§. Pro

§. Pro $w\sqrt{bcd^2+bdc^2}$. Poſito $d=1$.

1. Pro bc, ut $d=1$ ad db, ita c ad $\frac{bc}{1}=bc$.
2. $d^2=d$.
3. Med. Proport. inter bc & $d^2=d$, $=\sqrt{bcd^2}$
4. bc ut ante.
5. $dc=c$.
6. Med. proport. inter bc & $c=dc=\sqrt{bdc^2}$

Unde 7. habetur $\sqrt{bcd^2+bdc^2}$

Ac denique 8. Ope $d=1$, w quæſita. Quod erat propoſitum.



APPEN-

APPENDIX
PRO
Numeris irrationalibus

PROP. XXVI.

Ex dato numero quocunque radicem quadratam geometrice extrahere, juxta datam ſcalam unitatum.

Caſus 1.

Si datus numerus ſit compoſitus.

MEdia proportionalis inter partes componentes (quaſcunque, ſi propoſitus numerus plures diviſores agnoſcat) dabit radicem quæſitam.

EXEMPLA.

1. Pro $\sqrt{21}$. quær. Med. proport. inter 3 & 7.
2. Pro $\sqrt{30}$. quær. med. proport. inter 2 & 15, vel 3 & 10, vel denique 5 & 6.
3. Pro $\sqrt{99}$. quær. med. proport. inter 3 & 33, vel quod melius, inter 9 & 11.

Caſus

Caſus 2.

Si datus numerus ſit primus.

Dirimatur is in partes duas pluresve, prout res poſtulaverit, ita ut partes illæ, quantum fieri poſſit, quadrata rationalia, vel ſaltem rectangula, conſtituant, ac tum procedatur juxta 47. I. &c.

EXEMPLA.

Pro √ 11	dirimantur 11	in 4. 4. 3, vel 9 & 2
13	13	9 & 4
29	29	25 & 4
37	37	25 & 12
101	101	100 & 1 vel 81, 16, 4.

Porro in pleriſque numeris majoribus, ſive compoſiti ſint ſive primi, utraque operatio locum habere poteſt, ut patet ex ſequentibus exemplis.

1. Pro √700. Subtr. 700 ex 900, reſtant 200; quorum radice ex ſupra Caſ. 1. quæſitâ, & in ſemicirculo ex √900=30 ſubtractâ, habebitur quæſitum.

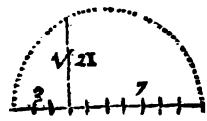
2. Pro √6000. Med. proport. inter 60 & 100 dabit √ quæſitam: Vel fiat diſtributio ut ad figuram conſtructionis ſubſequentem videre eſt.

3. Pro √4567. Diſtributio 1936 √ 44
2401 √ 49
4337 ſubtr.
4567
230 23
10

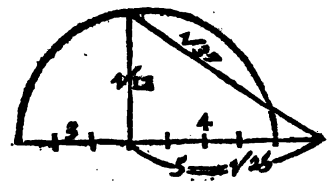
Sequuntur

Sequuntur Constructiones.

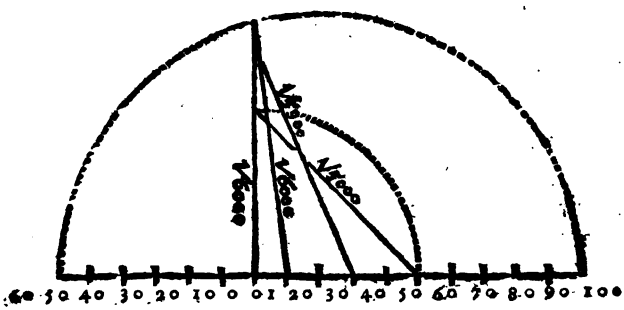
Pro √ 21.



Pro √ 37.



Pro √ 6000 | 100
6 0 |



Vel ita:

6000
50
50
2500
2
5000
30
30
900
10
10
100

add.

PROP.

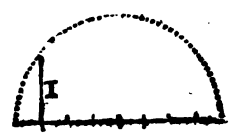
PROP. XXVII.

Data recta = radici cujuſcunque numeri dati, invenire rectam = unitati competenti ſeu numero alieni rationali.

SEcetur recta data in tot partes æquales, quot numerus datus unitates complectitur: Et jam quæratur media proport. inter totam lineam & ejus partem unam; vel inter dimidiam & partes duas, &c. quæ erit ipſa unitas quæſita.

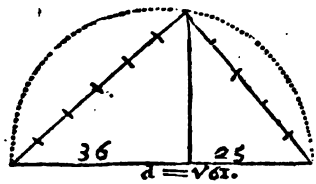
Vel, ſi numerus datus ſit magnus, ſecetur linea juxta eundem numerum in ea ratione, ut altera pars ſit = numero alicui quadrato: Jam inter totam lineam & partem quadraticam dictam quæratur med. proport. quæ erit √ ipſius quadratici numeri antea deſignati: Hâc itaque mediâ in tot partes æquales ſectâ, quot radix dicti numeri innuit, obtinebitur quæſitum.

1. Detur recta $a = \sqrt{7}$.



2. Detur

2. Detur recta $d = \sqrt{61}$



Secetur d in ratione ut 36 ad 25, quorum quidem uterque quadratus eſt; ac tum quæritur med. proport. inter totam & partem majorem, prodibit $\sqrt{36} = 6$; vel inter totam & partem minorem, proveniet $\sqrt{25} = 5$.

Prop. XXVIII. [Conſectaria præced.]

Data recta = radici cujuſcunque numeri dati, invenire rectam = radici cujuſcunque alterius numeri propoſiti, idque abſque ope unitatis.

Media proportionalis inter lineam totam & inter tot partes ejuſdem quot numerus quæſitæ innuit erit ipſa $\sqrt{}$ numeri propoſiti quæſita.

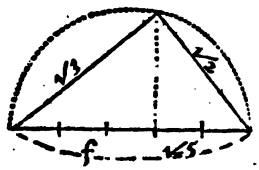
Oportet autem datam rectam in tot partes æquales ſecari, quot numerus datus, cujus radicem data recta infert, unitates continet (prout præced. Prop. dictum) Vel potius ea ratione ſectio facienda, ut altera pars ſit = alteri numero propoſito cujus nimirum radix expetitur.

Caſus

Caſus 1.

Ubi linea ſeu √ quæſita debet eſſe minor quam data.

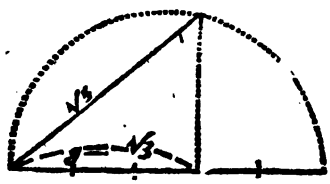
Detur recta $f = \sqrt{5}$; quæratur altera $= \sqrt{3}$.



Caſus 2.

Ubi linea ſeu √ quæſita debet eſſe major quam data.

Detur viciſſim $\sqrt{3} = g$; quæratur $\sqrt{5}$.



Æquationum Algebraicarum

Diſtributio ac Solutio

HOC EST,

De claſſibus ſeu Ordinibus Æquationum & ſingularum Claſſium Claſſibus ac formulis earumque reſolutione Geometrica.

ÆQuationum Algebraicarum tot ſunt Claſſes, quot gradus poteſtatum ſeu dimenſionum in ſcala quantitatum progreſſionali. Et tot dimenſionum dicitur aliqua æquatio, quot in quantitate incognita plurimæ reperiuntur; cæteris tamen paribus, hoc eſt, ſi & ſubſequentes gradus quantitatis incognitæ requiſiti adſint, adeoque ipſa æquatio terminis ſuis completa exiſtat: Secus enim per ſuccedentium terminorum, vel termini, abſentiam, æquationes ſæpe ad ſimpliciorem ſeu inferiorem Claſſem deprimuntur, &c.

CLAS-

CLASSIS I.

Æquationum ſimplicium ſeu unius Dimenſionis.

HArum duo ſunt Caſus:

1. Ubi x æquatur plano diviſo per lineam, vel quicquid huic æquipolleat.
2. Ubi x æquatur radici alicujus plani; vel etiam w alicujus planoplani.

Prioris Caſus: [Qui ſpectat quantitates abſolutas ſeu rationales] formulæ ſimpliciſſimæ ſunt

1. $x=\frac{a^2}{b}$ 2. $x=\frac{ab}{c}$

Quarum proinde ſolutio expeditur per tertiæ vel quartæ proportionalis inventionem, dicendo nimir.

1. Ut b ad a: ita a ad tertiam $= x$.
2. Ut c ad a: ita b ad quartam $= x$.

Poſterioris Caſus: [Qui concernit quantitates irrationales ſeu radicales] formulæ ſimpliciſſimæ ſunt

1.	$x^2=d^2$	Item	$x^4=d^4$
Unde	$x=d$		$x=d$
2.	$x^2=bd$		$x^4=b^2d^2$
Unde	$x=\sqrt{bd}$		$x=\sqrt{bd}$

Harum itaque ſolutio conſiſtit in lateris Quadrati inventione.

Cæterum in ſolvendis quæſtionibus æquationes rariſſimæ quidem in ejuſmodi ſimpliciſſima ſui forma emergere ſolent, ſed plerumque ex quantitatibus varie compoſitis affectiſque conflantur: quæ proinde ulterius expendendæ atque ad ſimpliciſſimam formulam reducendæ veniunt.

Sequuntur Exempla.

Quibus præcipuæ *Æquationum* dictarum (prout emergere ſolent) *variationes* ac *compoſita forma* continentur.

Et 1. quidem in *Quantitatibus abſolutis* ſeu *rationalibus.*

1. $bx = 2ac$ Unde $x = \frac{2ac}{b}$

Solutio: ut b ad $2a$: ita c ad x.
Vel: ut b ad $2c$: ita a ad x.

2. $3cx = b^2$ Unde $x = \frac{b^2}{3c}$

3. $2dx = 3a^2$ $x = \frac{3a^2}{2d}$

4. $cx + bx = 2b^2$ $x = \frac{2b^2}{c+b}$
Ut $c + b$ ad $2b$: ita b ad x.

5. $3ax + bx = 6ab$ $x = \frac{6ab}{3a+b}$

6. $dx - ax = 2a^2$ $x = \frac{2a^2}{d-a}$

7. $cx + x = d^2$ $x = \frac{d^2}{c+1}$

8. $2bx$

8. $2bx = a^2 + b^2$ Unde $x = \frac{a^2+b^2}{2b}$

9. $ax = ab + b^2$ $x = \frac{ab+b^2}{a}$

10. $2cx = b^2 - a^2$ $x = \frac{b^2-a^2}{2c}$

11. $3ax = 2bc + dc$ $x = \frac{2bc+dc}{3a}$

12. $dx + cx = ab + bc$ $x = \frac{ab+bc}{d+c}$

13. $2bx - ax = a^2 - \frac{1}{2}ab$ $x = \frac{a^2-\frac{1}{2}ab}{2b-a}$

14. $3ax - bx = 2b^2 - ac$ $x = \frac{2b^2-ac}{3a-b}$

15. $2cx = b^2 + c^2 - a^2$ $x = \frac{b^2+c^2-a^2}{2c}$

16. $ax - x = a^2 - b^2 - c^2$ $x = \frac{a^2-b^2-c^2}{a-1}$

17. $ax + cx - \frac{1}{2}dx = a^2 + c^2 - 2d^2$ $x = \frac{a^2+c^2-2d^2}{a+c-\frac{1}{2}d}$

18. $cx = b$ $x = \frac{b}{c}$

19. $bx = 4a$ $x = \frac{4a}{b}$

20. $x = ab$, ſeu $1x = ab$ $x = \frac{ab}{1}$

21. $2ax = a^2 - b$ $x = \frac{a^2-b}{2a}$

22. $3x = bc + d$ Unde $x = \frac{bc+d}{3}$

23. $a^2x + b^2x = ac + bc$ $x = \frac{ac+bc}{a^2+b^2}$

24. $dcx - x = 2d^2 + c$ $x = \frac{2d^2+c}{dc-1}$

25. $bdx = c^2a$ $x = \frac{c^2a}{bd}$

26. $c^2x = acd + a^2b$ $x = \frac{acd+a^2b}{c^2}$

27. $2bcx - adx = 3abc + bc^2$ $x = \frac{3abc+bc2}{2bc-ad}$

28. $bfx + dbx = efg$ $x = \frac{efg}{bf+db}$

29. $a^2x - d^2x = acd - d^2b$ $x = \frac{acd-d^2b}{a^2-d^2}$

30. $b^2dx = a^2c^2$ $x = \frac{a^2c^2}{b^2d}$

31. $2ac^2x - a^2bx = b^4 + 2b^2ac$ $x = \frac{b+2b^2ac}{2ac^2-a^2b}$

32. $4b^2x - adx = a^4 - c^4 - c^2a^2$ $x = \frac{a^4-b^4-c^2a^2}{4b^2-ad}$

2. *In Quantitatibus irrationalibus.*

33. $2x^2 = 3ab$ Unde $x = \sqrt{\frac{3ab}{2}}$

ſeu $x2\frac{3ab}{2}$

34. $x^2 = b^2 + ab$ $x = \sqrt{b^2+ab}$

35. x_2

35. $x^2 = b^2 - ab$

36. $4x^2 = 5a^2 - 3bc$

37. $bx^2 = 2a^2c$ $\qquad x = \sqrt{\frac{2a^2c}{b}}$

38. $ax^2 = 3b^2$

39. $bx^2 + x^2 = d^2$

40. $x^2 = a^2b$

41. $x^2 = d^3$

42. $ax^2 - 2bx^2 = ab$

43. $dx^2 = 2ac$

44. $3bx^2 = a^3 + ab^2$

45. $cx^2 + ax^2 = a^2c - b^3$

46. $b^2x^2 = 4a^3$

47. $acx^2 = b^2d^2$

48. $bfx^2 + dhx^2 = e^2gh$

49. $b^2x^2 - d^2x^2 = f^2d^2$

50. $a^2x^2 - \frac{1}{2}bdx^2 = d^4 + 2abd^2$

51. $x^2 = a^2bd$

52. $2ax^2 = d^4$

53. $x^4 = 3bcdf$ $\qquad x = \sqrt{3bcdf}$

54. $dx^4 + x^4 = a^2b^3$

55. $5x^4 = 8ad^3 + c^2ad$

56. $b^2x^4 = b^2c^2f^2 - 2a^6$

Ad Æquationes præcedentes Explicatio Caſuum quorundam.

PRimum quod ad uſum *Unitatis* attinet, in genere obſervandum, in omni æquatione requiri, ut Quantitates omnes ab utraque parte numero dimenſionum pares exiſtant; alias enim neque rationem neque æqualitatem ad invicem habebunt. Secus itaque ſi eveniat, hoc eſt ſi occurrat Æquatio quæ ab una parte pauciores literas ſeu dimenſiones præferat quam ab altera, aut ubi una quantitatum à communi dimenſionum numero deficiat, tunc ſemper defectum iſtum adſcitâ unitate, eâque in Quantitatem deficientem ducta quoties opus, ſuppleri intelligendum eſt: unde ſi una duntaxat Litera ſeu Dimenſio abſit, unitas illa lineam referet; ſi duæ, Quadratum; ſi tres Cubum. Hoc itaque pacto Æquationes ejuſmodi dimenſionum numero coæquandæ ſunt, antequam geometrice ſolvi poſſint. Et hoc quidem quoties uſu venerit, indicio eſt Problema eſſe in ſe Arithmeticum, nec niſi per numeros determinari vel exprimi poſſe: unde neceſſe erit, ut datis literis ſeu lineis, certi numeri, ſuus cuique, aſſignentur, quo quidem ipſa ſimul unitas [hoc eſt linea unitatem referens] determinabitur. Aut ſi unitas aſſumatur pro lubitu, tunc ſemper quantitates datæ quoad valorem numericum ab aſſumptâ unitate dependebunt, ac conſequenter ipſum quæſitum; quod proinde pro alia atque alia unitate variabitur, ut nunc majus nunc minus emergat.

Vel

Vel etiam ita hanc rem exprimere licebit; quum nimirum, uti dictum, ejuſmodi Problemata in ſua natura ſint arithmetica, & vero faciendum, ut geometrice ſolvi atque exponi poſſint, neceſſe erit ut Quantitates numericæ ad formam linearem reducantur; quo quidem fiet ut ipſa quoque unitas ſit certa quædam linea; Quæ proinde in alteram lineam numeri alicujus interpretem ducta Quadratum conſtituet; quod tamen Quadratum non niſi poteſtate lineari gaudebit. Vid. Prop. V. Logiſt. Linear

Num. 7. $cx+x=d^2$; hoc eſt, per obſervation. præced. $cx+1x=d^2$; unde $x=\frac{d^2}{c+1}$

Diviſor $c+1$ arguit problema eſſe numericum. Unde vel (1.) Quantitates c & d, ſi ſint lineæ, non niſi ſub certis numeris dari vel aſſumi poſſunt, quo tunc ipſa ſimul unitas innoteſcet; Quæ ſi vocetur e, exemplum ſic ſtabit $cx+ex=d_2$, &c. Aut (2.) ſi unitatem velimus aſſumere pro lubitu, tunc ipſa illa aſſumpta unitas lineis c & d ſuos numeros determinabit, &c.

Num. 8. $2bx=a^2+b^2$. Et $x=\frac{a^2+b^2}{2b}$

Unde (1.) poſito $a^2+b^2=f$. Seu $x=\frac{a^2}{2b}+\frac{1}{2}=f$. [per XVII. Logiſt. Lin.] erit. ut $2b$ ad f ita f ad x:

Vel (2.) ut $2b$ ad a: ita a ad $\frac{a^2}{2b}$, adde $\frac{1}{2}b$, habebitur x.

Num. 9.

Num. 9. $ax = ab + b^2$. Et $x = \frac{ab+b_2}{a}$

Seu $x = b + \frac{b^2}{a}$

Quare (1.) posito $ab + b^2 = g^2$, erit.
Ut a ad g: ita g ad x.

Vel (2.) ut a ad b: ita b ad $\frac{b^2}{a}$;

summa erit $= x$.

Vel potius (3.) cum Quantitas $ab + b^2$ sit orta ex multiplicatione $a+b$ in b, —— fiet.

Ut a ad $a+b$: ita b ad $\frac{ab+b^2}{a} = x$.

Num. 10. $2cx = b^2 - a^2$. Et $x = \frac{b^2-a^2}{2c}$

1.) $b^2 - a^2 = g^2$. Unde, ut $2c$ ad g: ita g ad x.

2.) Ut $2c$ ad b: ita b ad $\frac{b_2}{2c} = d$
Item, Ut $2c$ ad a: ita a ad $\frac{a^2}{2c} = f$
} unde $d - f = x$.

3.) Advertendo Quantitatem $b^2 - a^2$ esse compositam per multiplicationem $b+a$ in $b-a$; erit ut $2c$ ad $b+a$: ita $b-a$ ad $\frac{b^2-a^2}{2c} = x$.

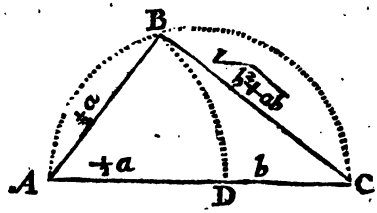
Num. 34. $x^2 = b^2 + ab$. Et $x = \sqrt{b^2 + ab}$.

1.) Posito $ab = d^2$ [per XVI. Logist. Lin.] erit $x = \sqrt{b^2 + d^2}$

Vel

Vel 2.) Observato Quantitatem b^2+ab esse oriundam ex multipl. $b+a$ in b, proportio erit, ut $b+a$ ad x: ita x ad b. Unde med. proport. inter b & $b+a=x$.

Vel 3.) Construatur triang. rectang. ABC, cujus Hypotenusa AC sit $=\frac{1}{2}a+b$ [nimirum AD $=\frac{1}{2}a$, & DC$=b$] & alterutrum latus circa rectum [ut hic AB] $=\frac{1}{2}a$; tunc enim alterum latus circa rectum, ut BC, erit $=\sqrt{b^2+ab}=x$.

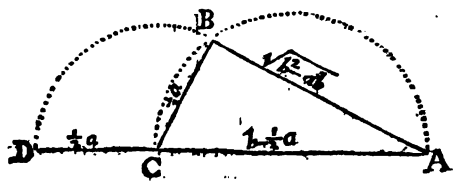


Num. 35. $x^2=b^2-ab$. Et $x=\sqrt{b^2-ab}$.

1.) Posito $ab=d^2$, erit $x=\sqrt{b^2-d^2}$.

2.) Ut $b+d$ ad x: ita x ad $b-d$. Unde med. proport. inter $b+d$ & $b-d=x$.

3.) Fiat in triangulo rectang. ABC Hypotenusa $AC=b-\frac{1}{2}a$, & $CB=\frac{1}{2}a$, erit $AB=\sqrt{b^2-ab}=x$.



§. $x^2 = b$. Unde $x = \sqrt{b}$.

Hoc loco vel quantitas numero determinanda eſt, quo innoteſcat unitas; vel ſi determinetur unitas, ab ea dependebit quantitas b, &c.

N°. 42. $a^2x^2 - 2abx^2 = a^2b^2$. Unde $x = \sqrt{\frac{a^2b^2}{a^2-2ab}}$

Vel $ax^2 - 2bx^2 = ab^2$. Unde $x = \sqrt{\frac{ab^2}{a-2b}}$

Seu ponendo $a - 2b = d$; $x = \sqrt{\frac{ab^2}{d}}$

1. *Modus.* Ponatur $\sqrt{a^2 - 2ab} = f$; erit.

Ut f ad a: ita b ad $\frac{ab}{f} = x$.

2. *Modus.* $x = \sqrt{\frac{ab^2}{d}}$

Ut d ad b: ita b ad $\frac{b^2}{d} = g$. unde $a = g\frac{ab^2}{d}$

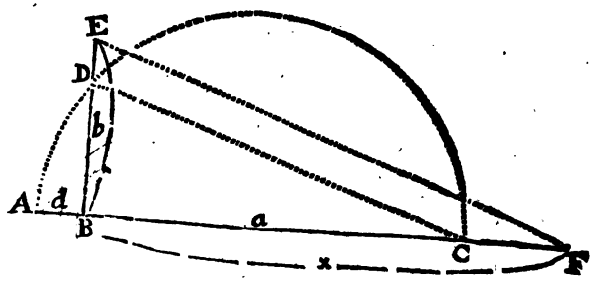
Conſequenter $x^2 = ag$. Et $x = \sqrt{ag}$.

3.) *Modus.* $dx^2 = ab^2$. Unde proportio.

Ut d ad a: ita b^2 ad x^2.

Cujus *Conſtructio:* Fiat [in appoſita Figura] AB $= d$ [hoc eſt $= a - 2b$] & BC $= a$ [hoc eſt addantur d & a] & deſcribatur ſuper AC ſemicirculus. Porrò ex B erigatur perpendic. BE $= b$ (hoc eſt $\sqrt{b^2}$) occurrens peripheriæ in D, tum ducta DC, & huic parallela EF ad prolongatam AC, erit BF $\sqrt{\frac{ab^2}{d}} = x$.

Demon-



Demonſtratio. Ut AB & BC [hoc eſt *d* ad *a*] ita ▭ BD ad ■ BC, per *Coroll.* 19. VI. ſed ut ■ BD ad ■ BC, ita & ■ BE [$=b^2$] ad ■ BF [$=x^2$] per 4. & 19. VI. Quare ■ BE (hoc eſt b^2) ad ■ BF, ut AB ad BC [hoc eſt ut *d* ad *a*] ideoque [per ſupra expoſitam proportionem] ■ BF$=x^2$, & conſequenter BF$=x$.

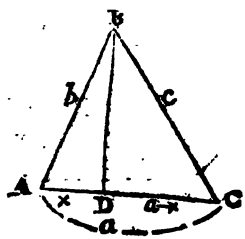
§. Vel brevius ita: BD$=\sqrt{ad}$, & ut BD ad BC: ita BE ad BF. Hoc eſt ut $\sqrt{ad}$ ad a: ita b ad x, unde $x\sqrt{ad}=ab$, & $x^2ad=a^2b^2$, & $x^2d=ab_2$: tum $x=\frac{ab}{\sqrt{ad}}$ ſed $\sqrt{ad}=\sqrt{a^2-2ab}$, quoniam $\frac{d}{ab}$ poſitum $=a-2b$; redit itaque æquatio $x=\sqrt{\frac{ab}{a^2-2ab}}$, quæ erat initio propoſita.

S E-

SEQUUNTUR

Quæstiones seu *Problemata*, Nonnulla.

1. DAtis in Triangulo [qualicunque] ABC lateribus, invenire perpendiculum BD [interius cadens].



Sit AC = a = 14
AB = b = 13
BC = c = 15
AD = x
∴ DC = a — x

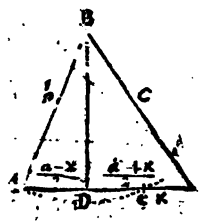
Jam 1.) In triangulo ABD
$b^2 - x^2$ = quadrato BD
2.) In triangulo BCD
$c^2 - a^2 + 2ax - x^2$ = quadrat BD
Unde $c^2 - a^2 + 2ax - x^2 = b^2 - x^2$
Et $2ax = b^2 + a^2 - c^2$
Div. $2a$

$$x = \frac{b^2 + a^2 - c^2}{2a}$$

Idem

Idem aliter?

Poſitis cæteris ut ſupra deſcribatur ex B arcus *Ae*.
Fiatque $eC = x$; erit
Igitur $DC = \frac{a+x}{2}$
Et $AD = \frac{a-x}{2}$



Jam 1.) In triangulo ABD.
$\frac{b^2 - a^2 + 2ax - x^2}{4}$ = BD quadrat.
2.) In triangulo BCD.
$\frac{c^2 - a^2 - 2ax - x_2}{4}$ = BD quadr.

Unde $\frac{b^2 - a_2 - x^2 + 2ax}{4} = \frac{c^2 - a^2 - 2ax - x_2}{4}$

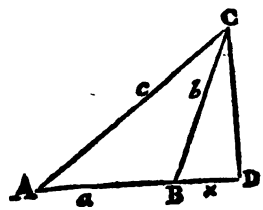
Et $4ax = c^2 - b^2$

Hoc eſt $ax = c^2 - b^2$

$x = \frac{c^2 - b^2}{a}$

2. Da-

2. Datis in triangulo obtuſangulo ABC lateribus, invenire perpendiculum BD [exterius cadens.]



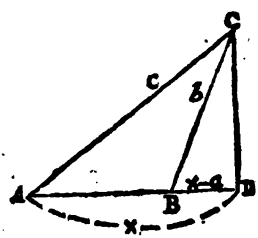
Sit $AB = a = 11$
$BC = b = 13$
$AC = c = 20$
$BD = x$

Jam 1.) In triangulo BCD.
$b^2 - x^2 =$ quadr. CD.
2.) In triangulo ACD.
$c^2 - a^2 - 2ax - x^2 =$ quadr. CD.

Quare $b^2 - x^2 = c^2 - a^2 - 2ax - x^2$
Et $2ax = c^2 - a^2 - b^2$
Div. $2a$

$$x = \frac{c^2 - a^2 - b^2}{2a}$$

Idem aliter.



Poſitis cæteris ut ſupra.

Sit nunc $AD = x$.
Erit $BD = x - a$.

Jam

Jam igitur 1.) In triangulo ACD
$c^2 - x^2 =$ quadr. CD
2.) In triangulo BCD
$b^2 - x^2 + 2ax - a^2 =$ quadr. CD.

Unde $b^2 - x^2 + 2ax - a^2 = c^2 - x^2$
Et $2ax = c^2 + a^2 - b^2$
Div. $2a$
$$x = \frac{c^2 + a^2 - b^2}{2a}$$

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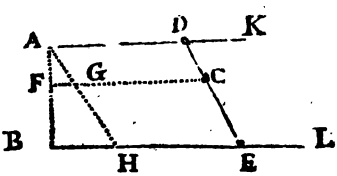
3. Datis duabus rectis AK, BL, parallelis, per datum intra ipſas punctum C rectum ducere DCE, quæ ad data puncta A & B in datis parallelis intercipiat ſegmenta AD, BE, in ratione data.

Sit $AF = a$
$FC = b$
$FB = c$
$AD, GC, HE = x$
$\therefore FG = b - x$



Sitque data ratio a ad f
10 18

Quoniam enim dantur duo puncta A & B, licebit rectam AB ducere, & CF parallelam ad AD vel BL; uti & AH ad ſuppoſitam DE queſitam.

Jam propter triangula AFG, ABH, similia.

AF FG : AB BH

$a \dots b - x : a + c \mid \frac{ab + bc - ax - cx}{a} = BH$

add x=HE, erit $\frac{ab + bc - ax - cx + ax}{a}$

Hoc est $\frac{ab + bc - cx}{a} = BE.$

Inde jam pro obtinenda æquatione.

ut data ratio : ita AD ad BE

$a \dots f : x \dots \frac{ab + bc - cx}{a}$

provenit $fx = ab + bc - cx$

& $fx + cx = ab + bc$

unde $x = \frac{ab + bc}{f + c}$

§. Erit itaque proportio æquationis.

ut $f + c$ ad $a + c$: ita b ad x

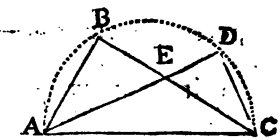
Seu rectius hoc ordine:

ut $c + f$ ad b: ita $c + a$ ad x.

4. Datis

4. Datis duobus triangulis ſuper eadem baſi inque eodem circuli ſegmento, ut ABC, ADC, invenire ſegmenta mutua laterum, AD, CB.

Sit $AB = a$
$BC = b$
$AD = c$
$DC = d$
$EC = x$
$\therefore BE = b - x$



Jam propter triangula ſimilia ABE, CDE.
ut AB ad BE : ita CD ad DE
$a \therefore b - x : d \mid \frac{bd - dx}{a}$

Iterum propter eadem
ut CD ad CE : ita AB ad AE
$d \therefore x : a \mid \frac{ax}{d}$

Jam ſi ex $AD = c$ ſubtrahatur $AE = \frac{ax}{d}$

remanet $DE = c - \frac{ax}{d}$ hoc eſt $\frac{cd - ax}{d} = DE$
ſed & DE ſupra inventa $= \frac{bd - dx}{a}$

quare $\frac{bd - dx}{a} = \frac{cd - ax}{d}$

& porro $bdd - ddx = acd - aax$
$aax - ddx = acd - bdd$
unde $x = \frac{acd - bdd}{aa - dd}$

Jam [pro conſtructione] ponendo $aa = df$.

erit $x = \frac{acd - bdd}{df - dd} = \frac{ac - bd}{f - d}$

porro ponendo $ac - bd = g^2$, & $f - d = b$

erit $x = \frac{g^2}{b}$

Atque ita propoſita æquatio ad formam ſimpliciſſimam eſt reducta.

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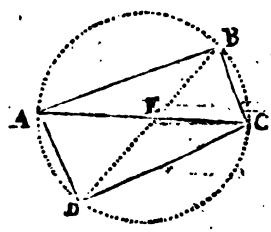
5. Datis duobus triangulis ſuper eadem baſi, & in ſegmentis circuli oppoſitis, ut ACB, ACD, ductâque ac data diagonio BD, invenire ſegmenta AE, EC; BE, ED.



Sit $AB = a$
$AC = b$
$BD = c$
$CD = d$
$EC = x$
$\therefore AE = b - x$

Jam

Jam propter triangula ABE, DEC, simil.

1.) Ut DC ad CE : ita AB ad BE

$$d \mathrel{\overset{\cdot\cdot}{-}} x : \quad a \;\Big|\; \frac{ax}{d}$$

2.) Ut AB ad AE : ita DC ad DE

$$a \mathrel{\overset{\cdot\cdot}{-}} b - x : \quad d \;\Big|\; \frac{db - dx}{a}$$

Si jam ex BD$=c$ subtr. BE$=\frac{ax}{d}$ erit

$$DE = c - \frac{ax}{d} = \frac{cd - ax}{d}$$

Sed & $DE = \frac{db - dx}{a}$ ——— ———

Quare $\frac{db - dx}{a} = \frac{cd - ax}{d}$

Ac denique $x = \frac{acd - bdd}{aa - dd}$

Prorsus ut in præced.

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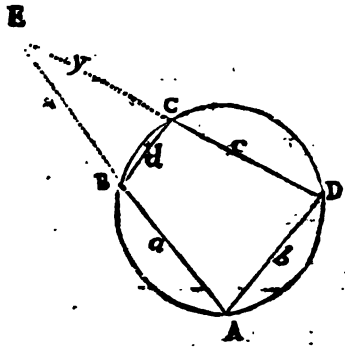
6. Datis quatuor lateribus, invenire circulum in quo illa quadrilaterum constituant.



Sit AB $= a$
AD $= b$
CD $= c$
BC $= d$
BE $= x$
CE $= y$

Difficultas quippe Problematis in eo consistit ut habeatur BE & CE.

Jam itaque propter triangula AED, CEB simil. erit 22. III.

1.) Ut AD ad AE : ita CB ad CE
$b \div a+x : d \div y$

Unde $by = ad+dx$
& $y = \frac{ad+dx}{b}$

2.) Ut AD ad DE : ita CB ad BE
$b \div c+y : d \div x$

Unde $cd+dy = bx$
& $y = \frac{bx-cd}{d}$

Quare

Quare $\frac{bx - cd}{d} = \frac{ad + dx}{b}$

Et porro $b^2x - bcd = ad^2 + d^2x$

$$b^2x - d^2x = ad^2 + bcd$$

$$x = \frac{ad^2 + bcd}{b^2 - d^2}$$

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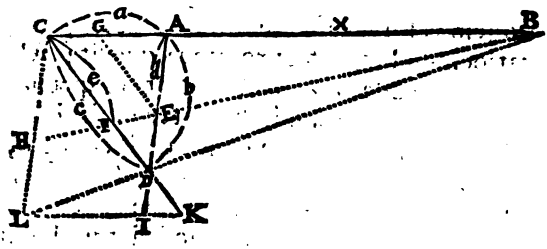
7. Invenire distantiam turrium, A, B, ex datis AC$=a$, AD$=b$, CD$=c$, AE$=d$, CF$=e$.



Esto AB $= x$

Ducta jam GE parallelâ ipsi CD erit propter triangula ADC, AEG, similia.

1.) Ut AD ad DC : ita AE ad EG.

$b \;\overline{\cdot\cdot}\; d : d \;\Big|\; \frac{cd}{b}$

2.) Ut AD ad AC : ita AE ad AG

$b \;\overline{\cdot\cdot}\; a : b \;\Big|\; \frac{ad}{b}$

add. AB $= x$ erit GB $= \frac{ad + bx}{b}$

Jam 3.) Propter simil. triangula CBF, GBE.
Ut CF ad CB : ita GE ad GB

$$e \mathrel{\overline{\cdot\cdot}} a+x : \quad \frac{cd}{b} \mathrel{\overline{\cdot\cdot}} \frac{ad+bx}{b}$$

$$\text{Unde } ade+bex = acd+cdx$$
$$\& \quad bex-cdx = acd-ade$$
$$\& \quad x = \frac{acd-ade}{be-cd}$$

Jam ut æqualitas hæc quam facillime in proportionem resolvatur, simulque eluceat, quo pacto ratiocinandum sit, pro linea quæsita AB$=x$, è datis obtinenda, advertere oportet, quænam litera plurimum omnium in hisce terminis seu quantitatibus reperiatur, quæ cum hoc loco sit d, utpote tribus in terminis occurrens, faciendum est ut illa etiam in reliquo [be] atque adeo in omnibus terminis inveniatur: Quod fiet si planum be commutetur in aliud cujus unum latus sit $=d$, dicendo nimirum

Ut d ad b: ita e ad alterum latus hujus plani, quod latus vocetur f unde $df = be$.

ac consequenter $\quad x = \dfrac{acd-ade}{df-cd}$

hoc est, eliso ubique d: $x = \dfrac{ac-ae}{f-c}$

porro posito $c-e=g$. & $f-c=h$.
erit $\quad x = \dfrac{ag}{h}$

adeoque ut h ad a: ita g ad x.

Idem

Idem aliter.

Poteſt enim eadem fractio $\frac{acd-ade}{be-cd}$ alia adhuc ratione reſolvi: conſiderando nimirum, in duobus terminis reperiri cd, & in reliquis duobus reperiri e: itaque ſi planum cd tranſmutetur in aliud cujus unum latus ſit e, litera e ſic in omnibus terminis habebitur ac proinde elidetur.

Fiat itaque, ut e ad c, ita d ad quartam quæ vocetur k, eritque jam $ek=cd$.

adeoque $$\frac{agk-adg}{bg-gk} \quad \frac{acd-ade}{be-cd}$$

hoc eſt $$\frac{ak-ad}{b-k}=x, \text{ &c.}$$

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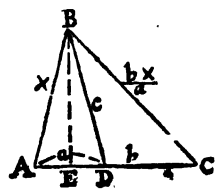
8. In triangulo ABC, angulo B ſecto bifariam per rectam BD, dantur AD, DC, & BD; quæruntur latera AB, BC.

Sit AD $= a$
DC $= b$
BD $= c$
AB $= x$



Jam

Jam 1.) per 3. VI. erit.

Ut AD ad DC : ita AB ad BC

$$a \;\overline{\cdot\cdot}\; b : \quad x \;\Big|\; \frac{bx}{a}$$

2.) per 13. II.

$$\blacksquare AD + \blacksquare BD - \square AB.$$

$$\frac{a^2 + c^2 - x^2}{2a} = DE$$

3.) per 12. II.

$$\blacksquare BC - \square BD - \square DC.$$

$$\frac{\frac{b^2x^2}{a^2} - c^2 - b^2}{2b} = DE$$

hoc est $\dfrac{b^2x^2 - a^2c^2 - a^2b^2}{2ba^2} = DE$

unde $\dfrac{b^2x^2 - a^2c^2 - a^2b^2}{2ba^2} = \dfrac{a^2 + c^2 - x^2}{2a}$

hoc est $\dfrac{b^2x^2 - a^2c^2 - a^2b^2}{ab} = a^2 + c^2 - x^2$

& $b^2x^2 - a^2c^2 - a^2b^2 = a^3b + abc^2 - abx^2$

tum $b^2x^2 + abx^2 = a^2b^2 + a^2c^2 + abc^2 + a^3b$

Et dividendo utrobique per $b + a$

$$bx^2 = a^2b + ac^2$$

unde $x^2 = \dfrac{a^2b + ac^2}{b}$

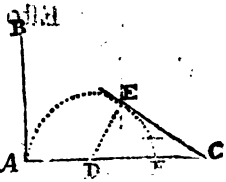
& posito $c^2 = bd$

$$x^2 = a^2 + ad$$

quare $x^2 = \sqrt{a^2 + ad}$

9. In

9. In triangulo rectangulo ABC deſcripto ſemicirculo AEF, tangente latera AB & BC, inveniendum ſit latus AB, ex datis AC & DE perpendiculari ad BC ex centro D.



Sit $AC = a$
$DE = b$
$AB = x$
$\therefore FC = a - 2b$

Jam 1.) Propter triangula ſimil. BAC, DEC.

Ut AB ad AC : ita DE ad EC.

$x : a : b \mid \frac{ab}{x}$

2.) Per 36. III. $\square EC = \square ACF$.

hoc eſt, $\frac{a^2b^2}{x^2} = a^2 - 2ab$

& $a^2b^2 = a^2x^2 - 2ba x^2$.

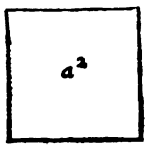
hoc eſt, $ab^2 = ax^2 - 2bx^2$.

$\frac{ab^2}{a-2b} = x^2$, &c.

Cujus reſolutio videatur ſupra inter formulas æquationum explicatas, *N°.* 42.

10. Sit

10. Sit quærenda linea, cujus Quadratum unà cum rectangulo sub tota, & ejusdem parte tertia sit æquale dato plano $= a^2$.



Esto linea desiderata $= x$

$\frac{1}{3} = b$

$\because \frac{1}{3}$ lineæ $= \frac{x}{b}$

Quare $x^2 + \frac{x^2}{b} = a^2$

$bx^2 + x^2 = ba^2$

Divid. $b + 1$

$x^2 = \frac{ba^2}{b+1}$

Et ponendo $1 = e$ erit

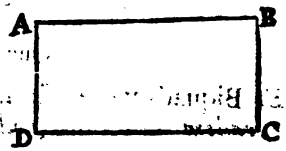
$bx^2 + ex^2 = ba^2$

$x^2 = \frac{ba^2}{b+e}$

11. Sit

11. Sit rectangulum AC, cujus longitudo AB dupla latitudinis AD, & ſumma Quadratorum ipſius AB, & AD, decupla ſummæ eorundem laterum ; quæruntur dicta latera AB, AD.

Ut Problema hoc generalius ſolvatur, ſit AD ad AB, ut e ad f; & □ AD + □ AB ad AD + AB, ut g ad e.



Eſto jam $AD = x$

erit $AB = \frac{fx}{e}$

& ſumma utriuſque $= \frac{ex+fx}{e}$

ſumma autem Quadratorum $= x^2 + \frac{f^2x^2}{e^2}$

ſeu $\frac{e^2x^2+f^2x^2}{e^2}$

unde nunc porro per alteram partem hypotheſeos erit, ut g ad e : ita $\frac{e^2x^2+f^2x^2}{e^2}$ ad $\frac{ex+fx}{e}$

unde $\frac{e^3x^2+ef^2x^2}{e^2} = egx+fgx$

hoc eſt $e^2x^2+f^2x^2 = egx+fgx$

div. x

$x = \frac{eg+fg}{e^2+f^2}$

CLAS-

CLASSIS II.

Equationum Quadraticarum, seu duarum Dimensionum:

Quo referuntur

Et Biquadraticæ *similiter affectæ, quippe quarum eadem ratio, nisi quod hic altera radicis extractio superaccedat.*

HUjus Classis Casus ac formulæ simplissimæ sunt tres:

1.) $x^2 = ax + b^2$
2.) $x^2 = -ax + b^2$
3.) $x^2 = ax - b^2$

Sic in *Biquadraticis.*

1.) $x^4 = a^2x^2 + b^4$
2.) $x^4 = -a^2x^2 + b^4$
3.) $x^4 = a^2x^2 - b^4$

Quarum singularum *Solutiones* ita se habent.

1.) $x = \sqrt{\frac{1}{4}a^2 + b^2} + \frac{1}{2}a$
2.) $x = \sqrt{\frac{1}{4}a^2 + b^2} - \frac{1}{2}a$
3.) $x = \frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 - b^2}$

Ubi not. Quod tertia hæc formula duas veras radices agnoscat, quarum altera major $x = \frac{1}{2}a + \sqrt{\frac{1}{4}a^2 - b^2}$, altera minor $x = \frac{1}{2}a - \sqrt{\frac{1}{4}a^2 - b^2}$

Utra

Utra vero in propoſita Quæſtione aliqua adhibenda ſit, ipſe uſus docebit; quia ſæpe utramque ſuo modo eidem conſtructioni inſervire poſſe animadvertetur.

Sic in *Biquadraticis.*

1.) $x^2 = \sqrt{\frac{1}{4}a^4 + b^4} + \frac{1}{2}a^2$

$x = \sqrt{\sqrt{\frac{1}{4}a^4 + b^4} + \frac{1}{2}a^2}$

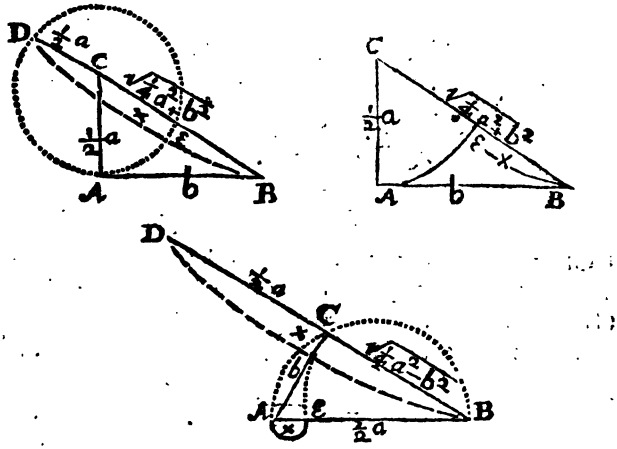
2.) $x^2 = \sqrt{\frac{1}{4}a^4 + b^4} - \frac{1}{2}a^2$

$x = \sqrt{\sqrt{\frac{1}{4}a^4 + b^4} - \frac{1}{2}a^2}$

3.) $x^2 = \frac{1}{2}a^2 \pm \sqrt{\frac{1}{4}a^4 - b^4}$

$x = \sqrt{\frac{1}{2}a^2 \pm \sqrt{\frac{1}{4}a^4 - b^4}}$

Porro *ſingulorum Caſuum Conſtructiones* inter cæteras ſunt iſtæ.

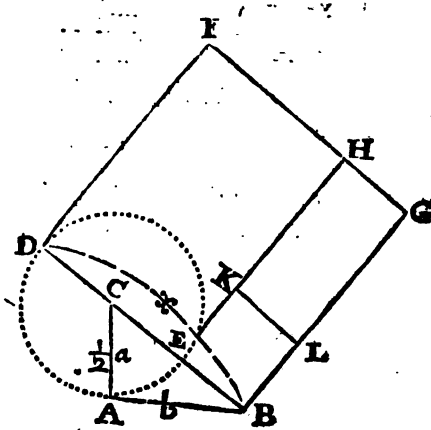


Quarum Explicatio & Demonſtratio vid. Pag. ſeq.

Demon-

Demonstratio Primo Casus.

Sit AC$=\frac{1}{2}a$, & AB$=b$: quibus conjunctis ad angulum rectum in A, describatur centro C, radio CA, Circulus ADE, ductâque recta DCB$=x$, fiat super eadem Quadratum BDFG$=x^2$; agatur porro EH, parallela ipsi BG.



Dico 1.) x^2 esse $=ax+b^2$. Nam primum DCE$=a$, & DF$=x$, componunt rectang. DH$=ax$; deinde per 36. III rectang. DBe [hoc est GBE] est=quadrat. AB,$=b^2$. Quare $ax+b^2=x^2$.

Dico 2.) x esse$=\sqrt{\frac{1}{4}a^2+b^2}+\frac{1}{2}a$. Nam Quadr. AC est$=\frac{1}{4}a^2$, & ▫ AB$=b_2$: quibus per 47. I æquatur ▫ BC; ergo BC$=\sqrt{\frac{1}{4}a^2+b^2}$. adde CE$=$CA $=\frac{1}{2}a$ erit BD $=$ BC $+$ CD. Hoc est $x=\sqrt{\frac{1}{4}a^2+b^2}-\frac{1}{2}a$. *q. e. d.*

Demonstratio

Demonſtratio Secundi Caſus.

Conſtructâ figurâ ut ante, ſit nunc $eB = x$; unde $DB = a + x$ deſcriptum autem ab eB Quadratum eL, $= x^2$.

Dico nunc 1.) x^2 eſſe $= b^2 - ax$. Nam $GB = DB = a + x$, & $LB = eB = x$; ergo $GL = De = a$; quæ cum $KL = eB = x$, conſtituit $\square$ $GK = ax$. Itaque $\square$ Ge [hoc eſt $\square$ $GK + \square eL$] $= x^2 + ax$. Quod ipſum $\square$ Ge, quum Caſu precedenti oſtenſum ſit $= \square AB = b^2$; erit $x^2 + ax = b^2$ & $x^2 = b^2 - ax$.

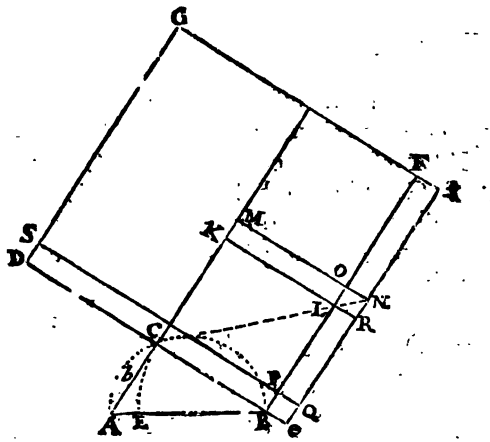
Dico 2.) x eſſe $= \sqrt{\frac{1}{4}a^2 + b^2} - \frac{1}{2}a$. Nam $Ce = \frac{1}{2}a$, & $eB = x$, unde $CB = \frac{1}{2}a + x = \sqrt{\frac{1}{4}a^2 + b^2}$ per præced. ideóque $x = \sqrt{\frac{1}{4}a^2 + b^2} - \frac{1}{2}a$. *q. e. d.*

Demonſtratio Tertii Caſus.

Sit in figurâ ſequenti, $AB = \frac{1}{2}a$, & $AC = b$. Deſcripto ſuper AB ſemicirculo, applicetur AC ad peripheriam, jungaturque BC: quâ prolongatâ fiat CD, $=$ AB.

Sit jam (1.) $DB = x$, tanquam radix major, ſuper qua conſtitutum $\square$ DF erit $= x^2$. Fiat porro $CE = CD = AB$, agaturque eH parallela ipſi BF, occurrens GE productæ in H, erit $\square$ $DH = ax$.

Dico 1.) x esse $= \frac{1}{2}a + \sqrt{\frac{1}{4}a^2 - b^2}$ per 47. 1 & constructionem.



Dico 2.) x^2 esse $= ax - b^2$. Constituto enim super $Ce = \frac{1}{2}a$ quadrato CN, erit illud $= \frac{1}{4}a^2$: itemque super CB $= \sqrt{\frac{1}{4}a^2 - b^2}$, descripto quadrato CL, erit hoc $= \frac{1}{4}a^2 - b^2$. Quare gnomon KNB erit $= b^2$: sed gnomoni huic æquatur $= \square$ BH, quod sic ostenditur: Ductâ diagonio CN, complementa ML, eL, erunt æqualia per 43. 1. sed ipsi eL est etiam $= \square$ OH: Si enim ex BF $=$ BD auferatur BO $=$ eN $=$ eC, remanebit OF $=$ BC, $=$ BL: Unde cum & OH sit $=$ OK, erit $\square$ BH $=$ gnomoni KNB $= b^2$; ac proinde $\square$ DF $= \square$ DH $- \square$ BH; hoc est, $x^2 = ax - b^2$. *q. e. d.*

Sit

Sit (2.) Be, $= AE$, $= x$, utpote minor: Super qua conſtitutum quadratum BQ erit $= x^2$, & $\square eS = ax$: ex quo ſi auferatur $\blacksquare BQ$, erit reliquum $\square BS = BH = b^2$. Unde jam reliqua facile patent.

Porro *compoſitæ Æquationum Quadraticarum ac Biquadraticarum formæ* ſunt fere hujuſmodi; eadem ratione, ut in prima Claſſe, ad ſimplices reducendæ.

1. $2x^2 = 4ax - a^2$. Et $x^2 = 2ax - \frac{a^2}{2}$
2. $x^2 = 2ax + 2bx - ab$.
   ponendo $2a + 2b = f$, erit
   $x^2 = fx - ab$
3. $2x^2 = 2ax + b^2 - a^2$.
   poſito $b^2 - a^2 = d^2$, erit
   $x^2 = ax + \frac{1}{2}d^2$
4. $ex^2 = 2adx + 2adf$
   $x^2 = \frac{2adx}{e} + \frac{2adf}{e}$ & poſito $\frac{2ad}{e} = g$
   $x^2 = gx + gf$.
5. $bx^2 = abx + ac^2$
   $x^2 = ax + \frac{ac^2}{b}$
   poſito $\frac{c^2}{b} = d$, erit $\frac{ac^2}{b} = ad$; unde
   $x^2 = ax + ad$

6. $3ax^2 = b^2x + ab^2$

$$x^2 = \frac{b^2x}{3a} + \frac{b^2}{3} \left| \frac{b_2}{3a} \right| = d;$$ unde

$$x^2 = dx + \tfrac{b^2}{3}$$

7. $bx^2 = -d^2x + abf - ad^2$

$$x^2 = -\frac{d^2x + abf - ad^2}{b} \left| \frac{d^2}{b} = g \right.;$$ unde

$$x^2 = -gx + af - ag \mid f - g = k$$

$$x^2 = -gx + ak,$$ &c.

8. $5x^2 = 9ax + 9bx + 2b^2 - 4\frac{1}{2}a^2$

$$\frac{9a + 9b}{5} = d \left| \frac{2b^2 - 4\frac{1}{2}a^2}{5} = \pm f^2 \right.$$

$$x^2 = dx \pm f^2$$

9. $ax^2 - bx^2 = a^2x + b^2$

$$a - b = d \left| \frac{a^2}{d} = f \right| \frac{b^2}{d} = bg;$$ unde

$$x^2 = -fx + bg$$

10. $4cx^2 = a^2x + b^2x - 2abx + 4c^2x + 4ab^2$

hoc loco advertendum $a^2 + b^2 - 2ab$ eſſe Quadratum rationale, cujus radix $a - b$, quæ ſi ponatur $= d$, erit

$4cx^2 = d^2x + 4c^2x + 4ab^2$, &

$$= \frac{d^2x}{4c} + cx + \frac{ab^2}{c}$$ porro

poſitis $\frac{d^2}{4c} = f \left| f + c = g \right| \frac{ab^2}{c} = ak$, erit

$x^2 = gx + ak.$

11. $x^2 = \frac{rkx + rm^2}{e+f+g+h} = n \left| \frac{rk}{n} = p \right| \frac{rm}{n} = q$; unde

$x^2 = px + mq$.

12. $b^2x^2 = -a^2cx + a^2b^2$

$x^2 = -\frac{a^2cx}{b^2} + a^2 \left| \frac{a^2}{b} = d \right| \frac{dc}{b} = f$; unde

$x^2 = -fx + a^2$

13. $b^2x^2 = 2ac^2x - a^2c^2$

$-d^2b - \frac{1}{4}d^4$, &c.

14. $x^3 = -\frac{2bc^2x + b^2c^2 + a^2d^2 - a^2c^2}{f^2}$

$\frac{2bc^2}{f^2} = g \left| \frac{b^2c^2}{f^2} = h^2 \right| \frac{a^2d^2}{f^2} = k^2$

$\frac{a^2c^2}{f^2} = l^2 \left| h^2 + k^2 - l^2 = m^2 \right.$; unde

$x^2 = -gx + m^2$

15. $b^2x^2 - a^2x^2 = 2a^2bx - 2b^2x + a^2b^2$

$a^2 = bd$; unde

$bx^2 - dx^2 = 2a^2x - 2b^2x + a^2b$

$b - d = f \left| \frac{2a^2 - 2b^2}{f} = g \right| \frac{a^2}{f} = h$; unde

$x^2 = gx + bh$

16. $a^2x^2 + b^2x^2 = f^2bx + a^2c^2 - \frac{1}{4}f^4$

$a^2 + b^2 = g^2$; unde

$x^2 = \frac{f^2bx + a^2c^2 - \frac{1}{4}f^4}{g^2}$

$\frac{f^2}{g} = h \left| \frac{a^2}{g} = k$; unde

$x^2 = \frac{hbx + kc^2 - \frac{1}{4}hf_2}{g}$

iterum $\frac{hb}{g} = l \left| \frac{c_2}{g} = m \right| \frac{f^2}{g} = h$; unde

$x^2 = lx + km - \frac{1}{2}h2$

$km - \frac{1}{2}h^2 = n^2$; unde

$x^2 = lx \pm n^2$

17. $d^2x^2 - c^2x^2 = -c^2bx + d^2a^2 - c^2a^2 + b^2c^2$

primò divid. $d^2a^2 - c^2a^2$ per $d^2 - c^2$

Quotus erit a^2, ponaturque $d^2 - c_2 = f_2$ erit

$x_2 = \frac{-c^2bx + b^2c^2}{f^2} + a^2$

porro $\frac{c^2}{f} = g$, erit

$x^2 = \frac{-gbx + b^2g}{f} + a^2$; tandem $\frac{bg}{f} = h$, erit

$x^2 = -hx + hb$

18. $b^2x^2 - 4x^2 = -8ax + 4a^2 + b^4$, &c.

Subjungi poterant Conſtructiones formarum Q. Quadraticarum, niſi quæ tum ſupra *Prop.* XXV. Logiſt. Lin. dicta ſunt, tum ſeq. Probl. dicentur, huic rei viderentur ſufficere.

Quæſtiones

Quæstiones seu *Exempla*

PRO

ÆQUATIONIBUS.

Quadraticis affectis.

Probl. 1. *Datum* a *dividere in partes* x *majorem & y minorem, ita ut productum sub partibus sit æquale dato* b.

Analysis. Quoniam 1° ex hyp. $x+y=a$ erit $y=a-x$. Et 2° ex hyp. $xy=b$, ergo erit $y=\frac{b}{x}$ ergo cùm $a-x=y$ $(y=)\frac{b}{x}$ erit $ax-x^2=b$, sive $x^2=ax-b$. Quare è præmissis erit $x=\frac{a}{2}+\sqrt{\frac{a^2}{4}-b}$. & $y=\frac{a}{2}-\sqrt{\frac{a^2}{4}-b}$.

Aliter. Quoniam $a-x=y$, multiplicato utrinque per x erit $ax-x^2=xy=$ per hyp. ipsi b. Quare $ax-x^2=b$, &c.

Aliter adhuc. Erit ex hyp. pars minor $a-x$, quæ si ducatur in majorem x erit productum $ax-x^2=$dato b ex hyp. &c.

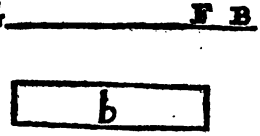
Nota. Iiſdem methodis tam numeri quam magnitudinis inveſtigantur.

Exemplum in Numeris.

Poſito aggregatum $a=20$, & productum $b=36$, & $x=$ majori numero quæſito, erit $20-x=$ minori, & proinde x in $20-x$, ſive $20x-x^2=36$, & proinde $x^2=20x-36$. Unde numeri quæſiti ex præmiſſis eruuntur, ſcil.

$$x=10+(\sqrt{100-36}=)\ 8=18$$
$$\&\quad y=10-(\sqrt{100-36}=)\ 8=2$$

Exemplum in Lineis.



Detur BG partiendum puta in F, ita ut rectangulum BF×FG ſit æquale rectangulo dato b.

Inv. Sit BG$=a$, majus Segmentum FG$=x$, erit ergo minus Segmentum BF$=a-x$, unde rectangulum ſub Segmentis x in $a-x$, ſive $xa-x^2=b$. Unde ut prius $x=\frac{a}{2}+\sqrt{\frac{a^2}{4}-b}$.

$$\&\quad y=\frac{a}{2}+\sqrt{\frac{a^2}{4}-b}.$$

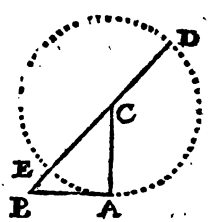
Quo-

Quorum omnium *Demonſtratio* primo ſit intuitu. Nam additis & multiplicatis quæ incognitis x & y inventa ſunt æqualia cognita & habebis ſummam & productum a & b, quod erat propoſitum.

Problema quidem facillimum, tyronum gratiâ, fuſius deducitur, tum ut iis minus impeditior foret aditus ad ſequentia, tum ut reliquis enucleandis lucem præbeat. Quâ de causâ Conſtructionem & Demonſtrationem ejuſdem Geometricam non importunum erit adjungere.

Conſtructio.

Fiat AB medium proportionale inter latera rectanguli b; quâ ut baſi, & hypoteneusâ $BC=\frac{GB}{2}$ fiat triangulum rectangulum ABC; poſtremo centro C, & radio CA deſcribe circulum AED.



Dico BD & BE eſſe Segmenta quæſita.

Demonſt. 1° Per Conſtr. $2BC=GB$. Patet autem $2BC=BD+BE$; ergo inventa BE & BD ſimul æqualia ſunt dato BG.

2° Per Conſtr. $BA^2=b$, atqui per (*Eucl.* III. 36) $BA^2=BE\times BD$, ergo rectangulum ſub inventis æquatur dato b. *Q. E. D.*

Probl.

Probl. 2. *Datis duarum quantitatum producto & differentia, easdem invenire singulatim.*

Probl. 3. *Datis duarum quantitatum differentia, & quadratorum earundem summa, quantitates invenire.*

Probl. 4. *Invenire duas quantitates, è datis earum producto, & Cuborum summa.*

Probl. 5. *E datis, trium continuo proportionalium summa, & summa quadratorum sub extremis proportionales invenire.*

Inv. Sint Prop. x, y, & z, quarum x maxima, proportionalium summa a, & quad. summa b. Quoniam itaque $x, y, z \div$ erit $y^2 = zx$ atqui ex hypothesi $x^2 + z^2 = b$, quare addito hinc $2y^2$ illinc $2zx$ erit $x^2 + 2zx + x^2 = b + 2y^2$, & proinde $x + z = \sqrt{b + 2y^2}$. Porrò ex hypoth. $a = y + (z + x =) \sqrt{b + 2y^2}$, unde $a - y = \sqrt{b + 2y^2}$, & $a^2 - 2ay + y^2 = b + 2y^2$, & $y^2 - 2ay - b + a^2$. Quare ex præmissis erit $y = \sqrt{2a_2 - b} - a$, &c.

Probl.

Probl. 6. *Datis trium continuo proportionalium altero extremo, & ſumma quadratorum ſub reliquis invenire proportionales.*

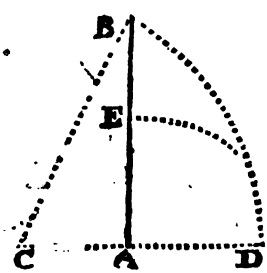
Sint a, x, y ÷ & quadratorum ſummâ b. Erit ex hyp. $ay = x^2$ ergo addito utrinque y^2. Erit $y^2 + ay = x^2 + y^2 = b$, ergo $y = \sqrt{\frac{a^2}{4} + b} - \frac{a}{2}$.

Probl. 7. *Datis ſummis, vel differentiis extremorum & mediorum quatuor continuo proportionalium, terminos ſingulatim inveſtigare.*

Probl. 8. *Dentur, aggregatum quatuor continuo proportionalium, & quadratorum ſub terminis mediis vel extremis. Eruendi ſunt ſinguli.*

Probl. 9. *Datam* AB *ſecare, ut quadratum partis* AE *rectangulo ſub tota & altera parte ſit æquale.*

Inventio. Poſitis AB $= a$, & AE $= x$, erit pars altera EB $= a - x$; & proinde ex hyp. $AE^2 = AB$ in EB, ſive $x^2 = a$ in $a - x = a - ax$ ideoque $x = \sqrt{\frac{a^2}{4}} - \frac{a}{2}$.



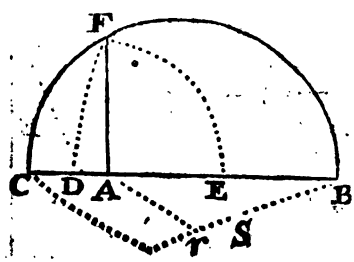
Conſtructio.

Constructio: Dimidia AB fiat perpendicularis ad eandem CA, & CD=CB, & AE=AD. Dico factum.

Demonstratio. $CA^2+AB^2=CB^2=CD^2=CA^2+(AD)\,EA\times(2CA)\,AB+(AD^2)\,AE^2$, ergo dempto utrinque CA^2+AE in AB erit $AE^2=AB^2-AE$ in $AB=AB\times(AB-AE)\,BE$. Ergo $AE^2=AB$ in EB. *Q. E. D.*

Probl. 10. *Datam* AB *secare ut quadratum partis* AE *ad rectangulum sub tota & altera parte rationem habeat datam* r *ad* s.

Inventio. Sit data $AB=a$
pars quæs. $AE=x$ erit
pars altera $EB=a-x$.
& ex hyp. AE^2, $AB\times EB :: r.\ s.$
i. e. x^2 (a in $a-x$) $a^2-ax :: r.\ s.$



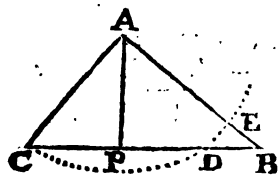
Constructio.

Conſtructio 1. Fiat ut in Schemate $s. r :: AB. AC = t.$ Ergo $AC = \frac{r}{s}$ in AB. Et $x^2 = ta - \frac{r}{s}x$.

Unde $x = \sqrt{ta + \frac{t^2}{4}} - \frac{t}{2}$

Conſtructio 2. Deſcripto ſemicirculo ſuper BA + AC diametro, ordinatim applicetur AF, & fiat DA = DC & DE = DF. Dico AE eſſe ſegmentum quæſitum.

Demonſtratio. $DA^2 + (AF^2)\ CA$ in $AB = DF^2 = DE^2 = DA^2 + AE^2 + (2DA)\ CA$ in AE. Ergo (CA in AB — CA in AE) AC in (AB — AE) $EB = AE^2$; atqui ex *Conſtr.* 1. $AC = \frac{r}{s}$ in AB, ergo $\frac{r}{s} \times$ AB in $EB = AE^2$, & proinde $r. s :: AE.$ AB in EB. *Q. E. D.*



Nota. In ſequent. Problemat. rectangulum ſupponi, cujus hypotenuſa notatur literis BC, latus majus BA, minus CA, perpendicularis in hypotenuſam AP, ſegmentum hypotenuſæ majus PB, minus CP & eorum differentia DB, differentia autem laterum EB.

Lem-

Lemma 1. Quadratum hypotenusæ æquari quadratis sub lateribus simul sumptis *Eucl.* I. 48. sive $BC^2 = BA^2 + CA^2$.

Lemma 2. Triangula similia, qualia sunt BAC, BPA, APC habere latera æqualibus angulis opposita proportionalia. *Eucl.* VI. *p.* 8. def. 1.

Probl. 11. *Datis differentia laterum* EB, *& perpendiculari in hyp.* AP, *invenire hypotenusam* BC, &c.

Analysis. Sit AP$=a$, EB$=b$, CB$=x$, & AC$=y$. Per *Lemm.* 1. erit $x^2 = y^2 + \overline{y+b}^2$ $y^2 + 2yb + b^2 = 2y + 2yb + b^2$. Ergo $x^2 - b^2 = 2y^2 + 2yb$

Et pr. *Lemm.* 2. BC . CA :: BA . AP.

sive $x \;.\; y \;::\; y+b \;.\; a$

ideoque $ax = y^2 + yb$, ergo $2ax = (2y^2 + 2yb =)$ $x^2 - b^2$. Unde $x^2 = 2ax + b^2$, & è præmissis $a + \sqrt{b^2 + a^2} = x$. *Q. E. I.*

Probl. 12. *Datis summa laterum* AB + AC, *& perpendiculari* AP: *invenire hypotenus.* BC, *&c.*

Probl. 13. *Invenire hyp. & Triangulum è datis differentia segmentorum* DB, *& latere alterutro.*

Probl.

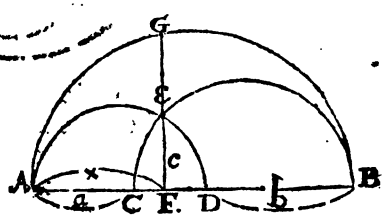
Probl. 14. *Conſtruendum ſit Triangulum, é datis differentia ſegmentorum* DB, *& ſumma laterum* AB + AC.

Probl. 15. *E differentiâ ſegmentorum* DB, *& differentia laterum* EB *datis, inveſtiganda ſunt reliqua.*

Probl. 16. *E latere alterutro & alterno & ſegmento eruere Triangulum.* Not. *Si detur* AB, *dicetur* PB *ſegment. alternum.*

Probl. 17. *Datis differentiis, inter hypotenuſam & latera, ſc.* CB—CA *&* CB—BA, *invenire ipſam hypotenuſam, &c.*

Probl. 18. *In ſubjecta figura dantur* AC = a, BD = b, FE = c, *quaruntur diametri ſingulorum circulorum.*



Sit AF = x

∵ CF = x—a

Inventio

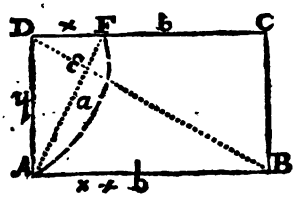
Inventio Æquationis facilis est.

provenietque $x^3 = ax + \frac{ac^2}{b}$

ac ponendo $\frac{c^2}{b} = d$, erit

$x^3 = ax + ad.$

Probl. 19. *In hoc rectangulo* ABCD, *ducta* AF *ad angulos rectos per diagonium* BD, *dantur* AF=a *&c.* FC=b; *quæruntur latera ipsius rectanguli.*



Sit DF = x
DA = y

Jam 1.) In triangulo ADF
$y^2 = a^2 - x^2$

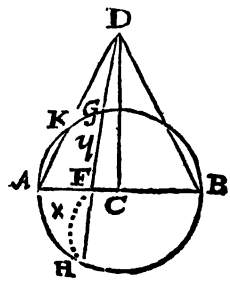
2.) Proport. △ ADF, BAD simil.
$y \cdot\cdot x : x + b \cdot\cdot y$

unde $y^2 = x^2 + bx$
Quare $x^2 + bx = a^2 - x^2$
$2x^2 = -bx + a^2$
$x^2 = \frac{-bx + a^2}{2}$
$x = \sqrt{\frac{1}{16}b^2 + \frac{1}{2}a^2} + \frac{1}{4}b.$

Probl.

Probl. 20. *Super basi trianguli aquilateri* ABD *descriptas est circulus, ductaque ex vertice* D *recta* DH *per punctum diametri datum* F, *secans peripheriam in* G; *quæritur longitudo ipsius secantis* DH, *ac singulorum ejus segmentorum, puta* DG, GF, FH; *supposito circuli Diametrum* AB *datum esse.*

Sit AC, AK, KD $= a$
CF $= b$
$\because$ AF $= a - b$
$\because$ BF $= a + b$
DC $[= \sqrt{3a^2}] = c$
$\therefore$ DF $= \sqrt{b^2 + c^2}, = d$
FH $= x$
FG $= y$
$\because$ DG $= d - y$



Jam 1.) Per 36. III.
$\square$ HDG $= \square$ ADK

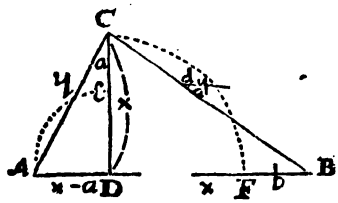
2.) Per 35. III.
$\square$ AFB $= \square$ HFG

Factis faciendis proveniet.

$$x^2 = \frac{3a^2x - b^2x - d^2x + a^2d - b^2d}{d}$$

tum ponendo $3a^2 - b^2 = f^2$
& $f^2 = dg$, erit
$x^2 = gx - dx + a^2 - b^2$, &c.

Probl. 21. *In triangulo* ABC, *demisso ex angulo* C *perpendiculo* CD, *datur differentia utriusque segmenti* AB *& perpendiculi* CD, *hoc est*, CE = a, *&* BF = b, *una cum ratione lateris* AC *ad* BC *ut* a *ad* d; *quæruntur latera trianguli* ABC.



$CE = a = 4$
$BE = b = 3$
$d = 6$
$CD = x$
$\because AD = x - a$
$\because BD = x + b$
$AC = y$
$\because BC = \frac{dy}{a}$

Jam 1.) In triangulo ACD
$y^2 = 2x^2 + a^2 - 2ax$

2.) In triangulo BCD
$$\frac{d^2y^2}{a^2} = 2x^2 + b^2 + 2bx$$
$$d^2y^2 = 2a^2x^2 + a^2b^2 + 2a^2bx$$
$$y^2 = \frac{2a^2x^2 + 2a^2bx + a^2b^2}{d^2}$$

Unde 3.) $2x^2 + a^2 - 2ax = 2a^2x^2 + 2a^2bx + a^2b^2 \div d^2$
$2d^2x^2 + a^2d^2 - 2d^2ax = 2a^2x^2 + 2a^2bx + a^2b^2$
$2d^2x^2 = 2a^2b + a^2b^2$
$-2ax + 2d^2ax - a^2d^2$
$d^2 = af$
$2afx^2 = 2a^2b + a^2b^2$
$-2aax + 2d^2ax - a^2d^2$

Divid.

Divid. per *a*.

$$2fx^2 - 2ax^2 = 2abx + 2d^2x + ab^2$$
$$2f - 2a = g \qquad - ad^2$$
$$x^2 = \frac{2abx + 2d^2x + ab^2 - ad^2}{g}$$

$$\frac{2ab}{g} = h \quad \left| \quad \frac{b_2}{g} = l \right.$$
$$\frac{2d^2}{g} = k \quad \left| \quad \frac{d^2}{g} = m \right.$$

$$x^2 = hx + kx + al - am$$
$$h + k = n \mid l - m = -p$$
$$x^2 = nx - ap$$

Conſtructiones per Numeros.

Pro *af*) $a \therefore d : d \therefore f$
$4 \quad 6 : 6 \mid 9$

Pro *g*) $18 = 2f$
$8 = 2a$
$10 = g$

§ $x^2 = \frac{2abx + 2d^2x + ab^2 - ad^2}{g}$

Æquatio hæc pertinet ad Caſum 3. quoniam ad^2 major quam ab^2.

$$\therefore x = \frac{ab + d^2}{g} + \sqrt{\frac{ab + d^2}{g^2}q + \frac{ab^2 - ad^2}{g}}$$

$\frac{ab+d_2}{g} = \frac{48}{10}$

$\because \frac{ab+d^2q}{g^2} = \frac{2304}{100}$

$\frac{ad^2-ab^2}{g} = \frac{108}{10}$

seu $\frac{1080}{100}$

2304
1080 subtr.
1224 √ 35 proximè
100 10

48 add.
10
83
10 = x. q. e. f.

Probl. 22. *Dentur duæ quantitates* = a & b; *quærenda sit tertia* = x, *ita ut ea addita ipsi* a, *summa sit Quadratum, addita vero eadem ipsi* b, *summa sit latus dicti quadrati: hoc est* a + x *quadratum*
b + x *ejusd. latus*

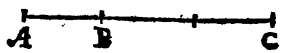
Unde $b^2 + x^2 + 2bx = a + x$.

Jam cum quantitas $a + x$ debeat esse Quadratum, erit ergo superficies per conseq. duarum dimensionum, adeoque in unitatem tanquam lineam ducenda. Assumpta igitur unitate $= b$, erit.

$$b^2 + x^2 + 2bx = ab + bx$$
$$\& \quad x^2 = -bx + ab - b^2$$
$$\text{unde} \quad x = \sqrt{\tfrac{1}{4}b^2 + ab - b^2} - \tfrac{1}{2}b$$
$$\text{hoc est} \quad x = \sqrt{ab - \tfrac{1}{4}b^2} - \tfrac{1}{2}b$$

Probl.

Probl. 23. *Datam rectam* AB, $= a$, *ita secare* [*extrema quadam & media ratione*] *ut Quadratum à parte majore, ad rectangulum sub tota & parte minore habeat datam rationem, ut* a *ad* f.



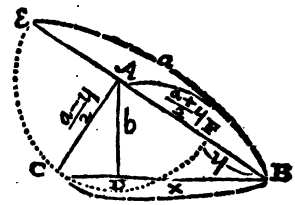
$$\overline{a^2 + x^2} - 2ax \qquad ax : a :: f$$

$$f a^2 + f x^2 - 2fax = a^2 x$$
$$f x^2 = a^2 x + 2fax - fa^2$$
$$a^2 = df$$
$$f x^2 = fdx + 2fax - fa^2$$
hoc est $$x^2 = dx + 2ax - a^2$$
$$d + 2a = 2g$$
$$x^2 = 2gx - a^2$$

Probl. 24. *In triangulo rectangulo* ABC *datur summa laterum circa rectum, hoc est* AB + AC, *cum altitudine* AD; *quæritur triangulum.*

Sit AB + AC $= a$
AD $= b$
BC $= x$
BF $= y$
∴ AB $= \frac{a+y}{2}$

∴ AC $= \frac{a-y}{2}$



Itaque 1.) In triangulo ABC.

$$\begin{array}{ccccc} \square AB & + & \square AC & = & \square BC \\ \dfrac{a^2+y^2+2ay}{4} & + & \dfrac{a^2+y^2-2ay}{4} & = & x^2 \end{array}$$

hoc est $\dfrac{2a^2+2y^2}{4} = x^2$ seu $\dfrac{a^2+y^2}{2} = x^2$

unde $y^2 = 2x^2 - a^2$

2.) Propter triang. simil. ABC, ADC

$$\frac{a+y}{2} \overset{\cdot\cdot}{=} x : b \overset{\cdot\cdot}{=} \frac{a-y}{2}$$

unde $bx = \dfrac{a^2-y^2}{4}$

& $y^2 = a^2 - 4bx$

Quare 3.) $2x^2 - a^2 = a^2 - 4bx$

$$2x^2 = -4bx + 2a^2$$
$$x^2 = -2bx + a^2$$
$$x = \sqrt{b^2+a^2} - b$$

Idem

Idem aliter.

Sit $BC = x$ | Itaque 1.) In triang. ABC.
$CA = y$ |

$$2y^2 = x^2 - a^2 + 2ay$$
$$y^2 = ay + \frac{x^2 - a^2}{2}$$

unde $y = \sqrt{\frac{1}{4}a^2 + \frac{1}{2}x^2 - \frac{1}{2}a^2} + \frac{1}{2}a$

hoc est $y = \sqrt{\frac{1}{2}x^2 - \frac{1}{4}a^2} + \frac{1}{2}a$

2. Propter triangula simil. ABC, ADC,

$$a - y \;\overset{\cdot\cdot}{-}\; x : b \;\overset{\cdot\cdot}{-}\; y$$

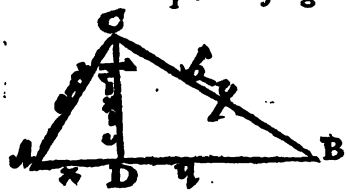
unde $bx = ay - y^2$
$$y^2 = ay - bx$$
$$y = \sqrt{\tfrac{1}{4}a^2 - bx} + \tfrac{1}{2}a$$

[Notanda hæc methodus duarum Æquationum ejusdem formæ; cujus usus uti pulcherrimus ita nonnunquam omnino necessarius est.]

Quare 3.) $\sqrt{\frac{1}{2}x^2 - \frac{1}{4}a^2} + \frac{1}{2}a = \sqrt{\frac{1}{4}a^2 - bx} + \frac{1}{2}a$
& abjecto signo radicali,
$$\tfrac{1}{2}x^2 - \tfrac{1}{4}a^2 = \tfrac{1}{4}a^2 - bx$$
$$\frac{x^2}{2} = -bx + \frac{a^2}{2}$$
$$x^2 = -2bx + a^2 \text{ ut supra.}$$

§. Sic etiam si detur differentia dictorum laterum, cum altitudine.

Probl. 25. *In hoc triangulo dantur* $\begin{cases} AC+AD=a \\ BC+BD=b \\ AB+CD=c \end{cases}$ *oportet distinguere singulas.*



Sit $AD=x$
$BD=y$
$\because\ AC=a-x$
$\because\ BC=b-y$
$\because\ DC=c-x-y$

Jam 1.) $\blacksquare AC-\boxtimes AD=\blacksquare BC-\blacksquare BD$

$a^2+x^2-2ax \qquad b^2+y^2-2by$
$-x^2 \qquad -y^2$
$a^2-2ax=b^2-2by$
$2by=b^2-a^2+2ax$
$y=\dfrac{b^2-a^2+2ax}{2b}$

2.) $\blacksquare AC-\blacksquare AD=\boxdot CD$

$a^2-2ax=c^2+x^2+y^2$
$-2cx-2cy+2xy$

Unde $y^2=2cy-2xy+a^2-c^2-x^2+2cx$
$-2ax$

& $y=c-x+\sqrt{a^2-2ax}$

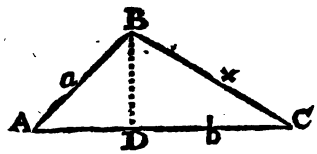
Quare 3.) $c-x+\sqrt{a^2-2ax}=\dfrac{b^2-a^2+2ax}{2b}$

$2bc-2bx+2b\sqrt{a^2-2ax}=b^2-a^2+2ax$
$2b\sqrt{a^2-2ax}=b^2-a^2-2bc+2ax$
$+2bx$

b^2-

$$b^2 - a^2 - 2bc = -2d^2$$
$$2a + 2b = 2f$$
$$2b\sqrt{a^2 - 2ax} = 2ax - 2d^2$$
$$b^2a^2 - 2b^2ax = f^2x^2 + d^4 - 2d^2fx$$
$$f^2x^2 = 2d^2fx - 2b^2ax + b^2a^2 - d^4, \text{ &c.}$$

Probl. 26. *In triangulo rectangulo* ABC *dantur* AB=a, CD=b, *quæruntur cætera.*



1.) $x^2 - b^2 = \square BD$
2.) $a^2 - x^2 + b^2 = \square AD$
3.) $\sqrt{a^2 - x^2 + b^2} + b = \sqrt{a^2 + x^2} = AC$

Unde prodibit

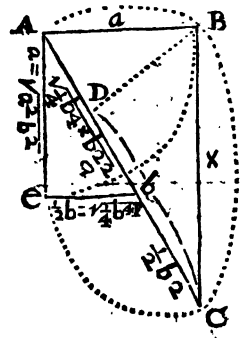
$$x^4 = b^2x^2 + b^2a^2$$
$$x^2 = \sqrt{\tfrac{1}{4}b^4 + b^2a^2} + \tfrac{1}{2}b^2$$
$$x = \sqrt{\sqrt{\tfrac{1}{4}b^4 + b^2a^2} + \tfrac{1}{2}b^2}$$

Not. Hæc quæstio faciliori quidem via solvi potest, si nimirum AD ponatur=x; At hic pro exemplo æqu. biquadr. esse debuit.

Itaque

Itaque pro Constructione.

Assumptâ b pro unitate, erit $b^2=b$, & $b^4=b$, & $\sqrt{b^2a^2}=a$, & $\sqrt{\frac{1}{4}b^4}=\frac{1}{2}b$; unde jam constructio facilis ut sequitur.



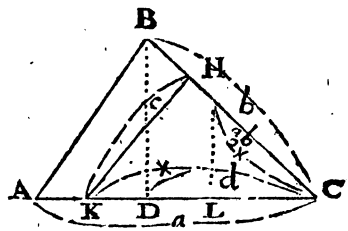
Probl.

Probl. 27. *Triangulum* ABC *dividere oportet in duo æqualia, ita ut linea biſectrix* KH *habeat datam longitudinem* [*poſſibilem, hoc eſt, quæ intra maximam & minimam conſiſtat.*]

Vel etiam proponatur ita:

In dato triangulo datam rectam [*idoneam*] *ita accommodare, ut ipſa triangulum ſecet in data ratione.*

Sit AC = *a*
BC = *b*
lin. data HK = *c*
CD = *d*
CK = *x*
∴ $CH = \frac{ab}{2x}$



Jam 1.) Pro CL per triangula BCD, HCL ſimil.

$$b \mp d : \frac{ab}{2x} \;\Big|\; \frac{abd}{2bx} = CL$$

$$\therefore \frac{2bx^2 - abd}{2bx} = KL$$

2.) $\square HK - \square KL = \square CH - \square CL$

$$4b^2c^2x^2 - 4b^2x^4 - a^2b^2d^2 + 4ab^2dx^2 = a^2b^4 - a^2b^2d^2$$

$$4b^2x^2$$

$$4b^2c^2x^2 - 4b^2x^4 + 4ab^2dx^2 = a^2b^4$$

Divid.

Divid. per $4b^2$

$$c^2x^2 - x^4 + adx^2 = \frac{abx}{4}$$

$$x^4 = c^2x^2 + adx^2 - \tfrac{1}{4}a^2b^2$$

$$c^2 + ad = f^2$$

$$x^4 = f^2x^2 - \tfrac{1}{4}a^2b^2$$

$$x^2 = \tfrac{1}{2}f^2 \pm \sqrt{\tfrac{1}{4}f^4 - \tfrac{1}{4}a^2b^2}$$

$$x = \sqrt{\tfrac{1}{2}f^2 \pm \sqrt{\tfrac{1}{4}f^4 - \tfrac{1}{4}a^2b^2}}$$

Pro *Constructione.*

Quæratur 1.) $\sqrt{ad}$ (2.) $\sqrt{ad + c^2} = f$. (3.) assumpta f pro unitate, erit $\frac{1}{2}f = \frac{1}{2}f^2 = \sqrt{\frac{1}{4}f^4}$. Quær. jam (4.) $\sqrt{\frac{1}{2}ab}$, indeque (5.) $\frac{\text{M}}{2}ab$ [dicendo: ut $f = 1$, ad $\sqrt{\frac{1}{2}ab}$: ita $\sqrt{\frac{1}{2}ab}$ ad $\frac{1}{2}ab$] $= \sqrt{\frac{1}{4}a^2b^2}$. (6.) $\sqrt{\frac{1}{4}f^4 \pm \frac{1}{4}a^2b^2}$ quæ (7.) add. & subtr. $\frac{1}{2}f = \frac{1}{2}f^2$. (8.) Ex summa & residuo extracta $\surd$ ope $f = 1$, dabit radicem æquationis utramque $= x$. Quarum major CM = CK; & minor CN = CH.

CLAS-

Classis III.

Æquationum Cubicarum ſeu trium Dimenſionum

HArum Caſus numerantur 14: qui tamen ſublatione ſecundi termini reducuntur ad tres vel quatuor, quos numeris ſuis ad marginem notavimus.

$x^3 = nx^2 + apx + aaq$
$x^3 = nx^2 - apx + aaq$
$x^3 = -nx^2 + apx + aaq$
$x^3 = -nx^2 - apx + aaq$
$x^3 = nx^2 + apx - aaq$
$x^3 = nx^2 - apx - aaq$
$x^3 = -nx^2 + apx - aaq$
1. $x^3 = * + apx + aaq$
2. $x^3 = * + apx - aaq$
3. $x^3 = * - apx + aaq$
$x^3 = nx_2 * + aaq$
$x^3 = nx^2 * - aaq$
$x^3 = -nx^2 * + aaq$
4. $x^3 = * * + aaq$

Æqua-

Æquationum Cubicarum & ultiorum ad solutionem præparatio.

Quæ fit 1:) Æquationem propositam ad simplicissimam Quantitatum formam reducendo.

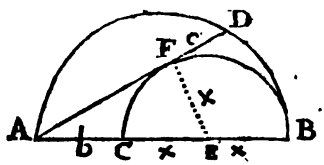
2.) Terminos omnes æquationis reductæ tam cognitos quam incognitos versus unam partem debite coordinando, atque adeo totam Æquationem ponendo=nihilo.

3.) Terminum secundum Æquationis sic coordinatæ tollendo.

4.) Æquationem hac operatione acquisitam correspondenti formulæ applicando, singulosque ejusdem terminos singulis istius coæquando.

Praxis dictorum exemplaris.

Probl. 25.] In hac figura dantur AC=b, FD =c; quæruntur cætera.



Posito BE = x

fiat

1.) EB ad FD : ut AE ad AF

$$x \mathrel{\overset{\cdot\cdot}{-}} c : \quad b+x \;\Big|\; \frac{bc+cx}{x}$$

2.)

2.) $\square AF + \square FE = \square AE.$

$$\frac{b^2c^2 + c^2x^2 + 2bc^2x}{x^2} + x^2 = b^2 + x^2 + 2bx$$
$$b^2c^2 + c^2x^2 + 2bc^2x = b^2x^2 + 2bx^3$$
$$2bx^3 = c^2x^2 + 2bc^2x + b^2c^2$$
$$\quad - b^2$$
$$x^3 = \frac{c^2x^2 + 2bc^2x + b^2c^2}{2b} - b^2$$

Sequitur 1. *Reductio ad ſimplicem formam.*

$$x^3 = \frac{-b^2 + c^2x^2 + 2bc^2x + b^2c^2}{2b}$$

ponendo $\frac{-b^2 + c^2}{2b} = -d$; & $\frac{1}{2}b = f$, erit

$$x^3 = -dx^2 + c^2x + fc^2$$

2.) *Coordinatio.*

$$x^3 + dx^2 - c^2x - fc^2 = 0$$

3.) *Secundi termini ſublatio.*

Ubi *Regula*: Ponatur $x = y$ [ſeu loco x aſſumatur y] + vel — quantitate cognita ſecundi termini, divisâ per numerum dimenſionum primi: + quidem ſi ſecundus terminus habeat —;

ac

ac viciſſim — ſi is habeat +, adeoque ſub affectione ſignorum contraria; atque tunc fiat quantitatum iſtarum ſubſtitutio conveniens, ut modo ſequetur.

§. Itaque in noſtro exemplo loco x ſumendum erit $y-\frac{1}{3}d$, cum ſecundus terminus ſit affectus ſigno +, & numerus dimenſionum primi termini ternarius.

Sequitur *Operatio.*

$$\left.\begin{array}{l} x = y - \frac{1}{3}d \\ x^2 = y^2 - \frac{2}{3}dy + \frac{1}{9}d^2 \end{array}\right\} \text{mult.}$$

$$\left.\begin{array}{l} y^3 - \frac{2}{3}dy^2 + \frac{1}{9}d^2y \\ -\frac{1}{3}dy^2 + \frac{2}{9}d^2y - \frac{1}{27}d^3 \end{array}\right\} \text{add.}$$

$$\left.\begin{array}{l} x^3 = y^3 - dy^2 + \frac{1}{3}d^2y - \frac{1}{27}d^3 \\ +dx^2 = \quad +dy^2 - \frac{2}{3}d^2y + \frac{1}{9}d^3 \\ -c^2x = \quad\quad -c^2y + \frac{1}{3}dc^2 \\ -fc^2 \quad\quad\quad\quad -fc^2 \end{array}\right\} \text{add.}$$

erit $y^3 * -\frac{1}{3}d^2y - c^2y + \frac{2}{27}d^3 + \frac{1}{3}dc^2 - fc^2 = 0$

ſeu $y^3 = * \begin{array}{l} +\frac{1}{3}d^2 \\ +c^2 \end{array} y \quad \begin{array}{l} -\frac{2}{27}d^3 \\ -\frac{1}{3}dc^2 \\ +fc^2 \end{array}$

Jam cum ultimus terminus conſtet diverſis ſignis ſeu quantitatibus partim negatis partim affirmatis, probe videndum eſt, utrum è dictis ſignis præpolleat, quod hoc loco deprehendetur eſſe ſignum +.

4. *Applicatio ad formulam.*

Sublatione ſecundi termini æquationes omnes reducuntur ad ſequentes tres formulas.

$$1.)\ y^3 = * + apy + aaq$$
$$2.)\ y^3 = * + apy - aaq$$
$$3.)\ y^3 = * - apy + aaq$$

Vel ſupponendo $a = 1$.

$$1.)\ y^3 = * + py + q$$
$$2.)\ y^3 = * + py - q$$
$$3.)\ y^3 = * - py + q$$

Cum itaque in acquiſita æquatione noſtri exempli bis recurrat ſignum $+$; facile colligitur eam correſpondere primæ formulæ. Quare jam erit ut ſequitur.

Termini ipſius formulæ. Termini Æquationis propoſ.

$$\begin{array}{lrcl}
 & y^3 & = & y^3 \\
 & +apy & = & +\frac{1}{3}d^2y + c^2y \\
 & \&\ ap & = & +\frac{1}{3}d^2 + c^2 \\
\text{unde} & p & = & \dfrac{\frac{1}{3}d + c^2}{a} \\
\hline
 & +aaq & = & -\frac{2}{27}d^3 - \frac{1}{3}dc^2 + fc^2 \\
\text{unde} & q & = & \dfrac{+fc^2 - \frac{1}{3}dc^2 - \frac{2}{27}d^3}{aa}
\end{array}$$

Sequitur *Præparationis* hactenus explicatæ *Constructio.*

§. Pro $\frac{-b^2+c^2}{2b}$; fiat $\frac{c^2-b}{2b} = -d$, quoniam b^2 major.

§. Pro $\frac{\frac{1}{3}d^2+c^2}{a}$; maxime ad rem erit obsequentia, si fiat $\frac{d^2}{a} = g$, & $\frac{c^2}{a} = h$; unde $h + \frac{1}{3}g = \frac{c^2+\frac{1}{3}d^2}{a} = p$.

§. Pro $\frac{d^2}{aa}$ (1.) ut a ad d, ita d ad $\frac{d^2}{a} = g$, quæ jam supra est inventa. (2.) Ut a ad d, ita g ad quartam $= k$, $= \frac{d^3}{aa}$.

§. Pro $\frac{dc^2}{aa}$ (1.) ut a ad c, ita c ad $\frac{c^2}{a} = h$, quæ jam supra est inventa (2.) ut a ad d, ita h ad quartam $= l$, $= \frac{dc^2}{aa}$.

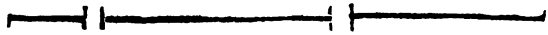
§. Pro $\frac{fc^2}{aa}$ [$f = \frac{1}{2}b$] cum hic iterum sit c^2, ut supra, erit $\frac{fc^2}{a} = fh$. (2.) Ut a ad f, ita h ad quartam $= m = \frac{fc^2}{aa}$

In de jam $m^2 - \frac{2}{27}k - \frac{1}{3}b = q$.

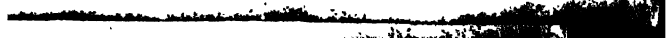
§. *Not.*

§. *Not.* Ad operationem commodius inſtituendam aſſumantur lineæ ſuperioris figuræ magnitudine dupla; ſitque $a =$ lineæ appoſitæ.

Paractâ operatione, provenient lineæ ad ſolutionem neceſſariæ ut hic videre eſt.



Fuit hactenus preparatio Æquationum Cubicarum, ſequitur dehinc ipſa Solutio.



ADDITAMENTA
AD
ALGEBRAM
JOHANNIS ALEXANDRI.

Per

HUMFREDUM DITTON,

Scholæ Novæ Mathematicæ

(Quæ est apud *Christi-Hospitium*)

MAGISTRUM.

LONDINI,

Typis excudit J. BARBER, *Hospitii* prædicti Typographus. MDCCIX.

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MONITUM AD LECTOREM.

CUM *Plurima, quæ multo ſunt Uſui in Praxi Algebraica & ad* Artis *tam profundæ, abſtruſiores receſſus, viam muniunt, In hoc Opuſculo vel fruſtra quæſiveris, vel carptim ab Authore Tractata inveneris; Idcirco Partem eorum quæ deſiderantur, ſequen-*

n, m, p, ſunt indices poteſtatum a^3, a^5, x^6, y^n, x^m, x^p, *&c.*

Ex hinc, viâ inversâ, facile intelligetur quid ſit *Radix*; quibus *evolutionibus* generari concipiendus eſt; & quomodo etiam *ſymbolice deſignandus*, vel exprimendus.

Sic, $x^{\frac{1}{2}}$, $x^{\frac{1}{3}}$, $x^{\frac{1}{4}}$, *&c.* Radices, *Quadratam*, *Cubicam*, Quadrato-quadratam, *&c.* Quantitatis x, denotant. Et $a^{\frac{2}{3}}$, ſignificat Radicem Cubicam Quadrati de a; & $y^{\frac{2}{5}}$, Radicem quintæ poteſtatis, Cubi ipſius y: & univerſaliter $x^{\frac{1}{n}}$, Radicem poteſtatis n de x. Ubi tamen litera n, ſignificare poteſt numerum quemlibet, integrum, vel fractum; & ſic $x^{\frac{1}{n}}$, *Perfectam*, aliquam ſive *Imperfectam* poteſtatem, aut Radicem quantitatis x repreſentare.

Et quidem notio *Poteſtatis* adeo generalis eſt, ut ad omnes quaſcunque expreſſionum diverſitates, extendatur.

Nec diſtinctio aliqua hic neceſſaria eſt, inter Radices & Poteſtates propriè ſic dictas; ſed ſolummodo mente concipiendum, quantitatem aliquam ad talem dignitatem elevari, qualem exprimit litera vel numerus ejus *exponens*, ſitne vel integer vel fractus. Sicut per $y^{\frac{n}{m}}$, intelligi debet quod quantitas y elevatur ad potentiam, cujus exponens deſignatur per $\frac{n}{m}$.

Jam Obſervandæ ſunt *Regulæ* ſequentes.

1. *Poteſtates aliquæ (ejuſdem quantitatis) in ſeſe multiplicantur, per Numerorum vel Literarum exponentium Additionem.*

Sic

Sic $a_3 \times a^5 = a^8$; $a^4 \times a^2 = a^6$; $x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{\frac{2}{2}} = x^1$; $d^n \times d^m = d^{n \times m}$; $x^{b \times 1} + x^{c \times 1} = x^{bc \times b \times c + 1}$, *&c.*

2. *Poteſtatum diviſio, fit per Numerorum vel Literarum exponentium Subſtractionem.*

Sic $x^7 —: x^5 = x^2$; $x^{\frac{7}{3}} —: x^{\frac{2}{3}} = x^{\frac{5}{3}}$; $y^3 —: y^8 = y^{-5}$; $a^{\frac{3}{2}} —: a^{\frac{7}{2}} = a^{-\frac{4}{2}} = a^{-2}$; $x^m —: x^n = x^{m-n}$; $y^{\frac{m}{n}} —: y^{\frac{p}{n}} = y^{\frac{m-p}{n}}$, *&c.*

Duæ Regulæ præcedentes, etiam hoc modo proponi poſſunt.

Si duarum Poteſtatum Indices, ſibi mutuo addantur, eorum ſumma, erit Index Poteſtatis, per Multiplicationem producta.

Et ſi Index Poteſtatis Dividentis, ab Indice Poteſtatis Dividendæ ſubtrahatur, eorum differentia, erit Index Poteſtatis ex Diviſione ortæ.

3. *Poteſtatum ad alias Poteſtates elevatio, fit per exponentium Multiplicationem.*

Vel hoc modo. Si poteſtas aliqua data, ad Poteſtatem aliam datam, elevanda ſit; oportet ut Index Poteſtatis propoſitæ, ducatur in Indicem Poteſtatis illius, ad quam prior debet aſcendere.

Sic x^5 ad Poteſtatem tertiam elevata eſt, $x^{5 \times 3}$, ſive x^{15}; ad Pot. 2^{dam} eſt $x^{5 \times 2} = x^{10}$; ad 7^{mam} eſt $x^{5 \times 7} = x^{35}$; y^n ad Pot. n, eſt y^{2n}; y^p ad Pot. q, eſt y^{pq}, *&c.*

4. *Om-*

4. *Omnigenarum Radicum extractio ſit per exponentium Diviſionem.*

Aut aliter. Si Radix aliqua elicienda ſit, Index Poteſtatis propoſitæ, dividendus eſt per Indicem Poteſtatis illius, cujus Radix requiritur.

Sic Radix *Quadrata*, ex a^8, eſt $a^{8-:2} = a^4$; *Cubica*, eſt $a^{8-:3}$, vel $a^{\frac{8}{3}}$; *Quadrato-Quadrata*; eſt $a^{8-:4}$, hoc eſt a^2. Sic Rad. $\frac{z}{5}$ ex y^4, eſt $y^{\frac{4}{2}\circ} = y^{10}$; Rad. m ex a^n, eſt $a^{\frac{n}{m}}$; Rad. n ex x^{2mn-nn}, eſt x^{2m-n}, *&c.*

Notandum tamen quod nuda multiplicatione, tum Poteſtatum elevatio, tum Radicum extractio, perfici poterit.

Adeo ut hæ duæ, optimè ad unum idemq; operationis Genus reducuntur. Nempe ſicut omnis Radix, ut Poteſtas quædam ſpectari poteſt; ſic etiam nil vetat quin Radicis extractio, ut elevatio quantitatis ad certam quandam Poteſtatem, conſideranda veniat.

Hoc modo Radix Cubica ex a^4, eſt $a^{4-:3}$, vel $a^{4\times\frac{1}{3}}$; & Rad. n, ex x^{3m}, eſt vel $x^{3m-:n}$, vel $x^{3m\times\frac{1}{n}}$ *&c.*

Nam eodem recidit, quantitatem unam per aliam dividere, ac per diviſoris *Reciprocam*, multiplicationem inſtituere.

In quantitatibus *Compoſitis*, eædem obtinent leges, quas hactenus in ſimplicibus oſtendimus.

Et harum omnium Rationes, ex *Poteſtatum Geneſi*, cuilibet rem attendenti, valde perſpicuæ erunt: Quod innuiſſe, ſatis eſt.

CAP.

CAP. II.

De Quantitatibus Surdis, *ſive* Irrationalibus.

NUmerorum Rationalium dantur quædam Radices, quæ in *Numeris Rationalibus* exprimi non poſſunt. Et tales Radices; *Surdæ vel Irrationales*, appellantur. Adeo ut per Quantitates *Surdas*, nil aliud volumus; quam Radices ejus Naturæ ac Indolis, ut omnimodam in Numeris Rationalibus expreſſionem reſpuant. Et quod de *Numeris* dictum eſt, etiam intelligi debet, de Quantitatibus, per *Species Algebraicas* deſignatis.

Communis nota Radicalitatis, eſt hæc $\surd$; adjuncto vel numero, vel literâ, Radicis naturam indicante, ſic $\surd(3)\,120$, vel $\sqrt[3]{\ }\,120$, denotat radicem cubicam numeri 120: $\surd(4)\,aab$ vel $\sqrt[4]{\ }aab$, Radicem biquadraticam, expreſſionis aab: & $\surd\overline{aaa+abb}$, Radicem Quadratam, quantitatis $aaa+abb$. Aliquando enim, plures termini, ſub uno eodemque ſigno Radicali concluſi ſunt, & tales Radices *Univerſales* vocantur; ſic $\sqrt[3]{\overline{ccd-n^3+m^3}}$, ſignificat Radicem Cubicam, Differentiæ inter $ccd+m^3$ & n^3, & $\sqrt[3]{p^3+\surd\overline{a^2b^3+c^4d^2}}$ Radicem Cubicam, quantitatis p^3, additæ Radici Quadratæ quantitatis a^2b^3

$b^3+c^3d^3$; & $\sqrt[4]{q^4+\sqrt[3]{a^3p^3+n^3m^3}-bb\sqrt{aa-cc}}$

Radicem Biquadraticam ex q^4, additam Radici Cubicæ, Differentiæ, quæ eſt inter $a^3p^3+n^3m^3$, & bb ductam, in Radicem quadratam quantitatis $aa-cc$. Et per hanc normam, de cæteris judicandum eſt.

Quantitates ſurdæ, ſi diverſa ſigna Radicalia habuerint, facillimè ad ſignum commune reducuntur, per vulgarem, cum Fractionibus *exponentibus*, operationem; faciendo nempe ut eæ ſub *eodem Denominatore* reperiantur.

Sic $\sqrt[3]{5}$ & $\sqrt[4]{7}$, hoc eſt $5^{\frac{1}{3}}$ & $7^{\frac{1}{4}}$, ſi Fractiones reducuntur ſunt $5^{\frac{4}{12}}$, & $7^{\frac{3}{12}}$, ſive $\sqrt[12]{5^4}$, & $\sqrt[12]{7^3}$, vel $\sqrt{(12)}625$, & $\sqrt{(12)}343$. Ad eundem modum, $\sqrt[2]{aq}$, & $\sqrt[3]{aaq}$, hoc eſt $aq^{\frac{1}{2}}$ & $aaq^{\frac{1}{3}}$, reductæ ſunt $aq^{\frac{3}{6}}$ & $aaq^{\frac{2}{6}}$, ſive $\sqrt[6]{aq^3}$, & $\sqrt[6]{aaq^2}$, vel $\sqrt{(6)}a^3q^3$, & $\sqrt{(6)}a^4q^2$. Ad vitandos verò errores, probè tenendum eſt, exponentem ſemper totam expreſſionem, & non ultimam ſolummodo literam (quam immediatè conſequitur) afficere. Ut ex. gr. per expreſſionem $\sqrt[6]{aaq^2}$, volumus ſextæ Poteſtatis Radicem, non ex $aaqq$, ſed ex Quadrato de aaq, hoc eſt a^4q^2. Et conſimilis in cæteris, conſtructio facienda eſt. Sit unum adhuc exemplum, quod pro omni operatione Numericâ, Regulæ loco erit. Proponantur $\sqrt[m]{a^q}$, & $\sqrt[n]{c^p}$, ſive $a^{\frac{q}{m}}$ & $c^{\frac{p}{n}}$, quæ factâ reductione, ſunt $a^{\frac{qn}{mn}}$, & $c^{\frac{mp}{mn}}$. Tum expoſitis Fractionibus $\frac{qn}{mn}$, & $\frac{pm}{mn}$, in minimis nu-

eæ

meris, *communem Denominatorem* habentibus, ſint eæ $\frac{r}{d}$ & $\frac{s}{d}$ reſpectivè. Adeoque erunt $a^{\frac{r}{d}}$ & $e^{\frac{s}{d}}$, ſive $\sqrt[d]{a^r}$, & $\sqrt[d]{e^s}$, expreſſiones quæſitæ, quæ ſigno radicali *communi & minimo*, fruuntur. Ex gr. ſi $\frac{q}{m} = \frac{1}{2}$, & $\frac{p}{n} = \frac{1}{4}$, tunc $\frac{q\,n}{mn} = \frac{4}{8}$, & $\frac{p\,m}{mn} = \frac{2}{8}$, ſicut $a^{\frac{q\,n}{mn}}$, & $e^{\frac{p\,m}{mn}}$ ſint $a^{\frac{4}{8}}$, & $e^{\frac{2}{8}}$, vel $\sqrt[8]{a^4}$ & $\sqrt[8]{e^2}$, hoc eſt (ſi ad infimos terminos perducuntur) $\sqrt[4]{a^2}$ & $\sqrt[4]{e^1}$, adeò ut d in hoc exemplo ſit 4.

Quantitas Rationalis ad ſignum aliquod Radicale reducitur, per *Involutionem* (ſive Multiplicationem in ſe) naturæ iſtius ſigni congruam.

Sic numerus 5 reducitur ad ſignum $\sqrt[3]{}$, per multiplicationem *Cubicam*, fitque $\sqrt[3]{125}$; & $e+x$ ad $\sqrt[2]{}$, multiplicando in ſe Quadratè, & tunc ejus loco ſcribitur $\sqrt[2]{e^2+2ex+xx}$; vel ſi ad $\sqrt[3]{}$, orietur, $\sqrt[3]{e^3+3e^2x+3ex^2+x^3}$. Et ſic de aliis. Non ergo pro rato haberi debet, expreſſionem quæ ſub ſigno aliquo radicali concluſa eſt, ſemper eſſe ſurdam; ſeu ejus generis, ut per quantitatem rationalem deſignari non poſſit. Cum ipſæ Quantitates Rationales, ad talem formam reducantur, & ſub hâc (ut ita dicam) Radicalitatis Larvâ, ſæpenumerò lateant. Hoc modo, $\sqrt{4}$, $\sqrt[3]{27}$, $\sqrt[4]{256}$, $\sqrt[5]{3125}$, &c. ſunt numeri naturales, 2, 3, 4, 5, &c. ut ex ſingulorum evolutione facile patet.

Radices ſurdæ non rarò ad *ſimpliciores* reducuntur, *dividendo* Quantitates ſub ſigno Radicali comprehenſas, per *Poteſtatem aliquam* ex quâ, per multiplicationem, fuerunt productæ. Sic

Sic $\sqrt{245xx}$, reducitur ad $7x\sqrt{5}$, quoniam $245xx = 49xx$ in 5, quarum Radices quadratæ ſunt $7x$ & $\sqrt{5}$. Eodem modo $\sqrt[3]{72e^3}$ evadit $2e\sqrt[3]{9}$, nam $72e^3$, producitur ex multiplicatione $8e^3$ in 9, quarum Radices Cubicæ ſunt $2e$, & $\sqrt[3]{9}$. Similiter pro $\sqrt{x^3b + x^2b^2}$, ſcribo $x\sqrt{xb + bb}$, quoniam facta diviſione ipſius $x^3b + x^2b^2$, per xx, oritur $xb + bb$. Et loco hujus, *viz.* $\sqrt{ad^2 + 2adx + axx + 2bd^2 + 4dbx + 2bx^2}$, ſubſtituitur, $d + x\sqrt{a + 2b}$, prior enim illa expreſſio, dividi poteſt per quadratum $d^2 + 2dx + x^2$, & quotiens eſt $a + 2b$.

Sic ob eandem rationem, loco $\sqrt{\frac{a^2n^2q^2}{r^2x^2} + \frac{8n^2q^2d}{rrx}}$, ſcribi poterit, $\frac{nq}{rx}\sqrt{aa + 8dx}$.

Si vero, inter Diviſores Quantitates Surdæ Propoſitæ, Talis Poteſtas inveniri nequeat, cujus ope Diviſio peragenda eſt; tunc ea, per hanc *Methodum*, ad formam ſimpliciorem, reduci non poterit. Sed tamen eam quodammodo exprimere potes, ſaltem ſi Fractionis Forma tibi non diſpliceat. *Ex. gr.* Sit $\sqrt{10aa}$; pro Fractione equivalenti, ſcribe $2a\sqrt{n}$, vel $3a\sqrt{n}$, vel $5a\sqrt{n}$, vel $10a\sqrt{n}$, vel a ductam in quemvis alium numerum ad arbitrium ſumptum. Tum ſi $\sqrt{10aa} = 2a\sqrt{n}$, erit $10aa = 4aan$, & $n = \frac{10}{4} = \frac{5}{2}$; ſic ut $\sqrt{10aa} = 2a\sqrt{\frac{5}{2}}$ ſi $\sqrt{10aa} = 5a\sqrt{n}$, tum $10aa = 25aan$, & $n = \frac{10}{25} = \frac{2}{5}$, & $\sqrt{10aa} = 5a\sqrt{\frac{2}{5}}$. Et ſic Infinitis modis, eandem Quantitatem Surdam deſignabis.

Notan-

Notandum eſt ulterius, Quantitatum Surdarum, alias *Commenſurabiles* exiſtere; hoc eſt, ſunt ad invicem, ut *Quantitas Rationalis*, ad *Quantitatem Rationalem*: alias verò *Incommenſurabiles*.

Et quomodo ſurda aliqua data, ſe habet ad aliam datam quoad *hoſce reſpectus*; ex ambarum ad ſimpliciſſimos terminos reductione, facilè patebit. Inveniatur nempe, *maximus utriuſque communis diviſor*, & judicium fiat de *Quotientibus*, ſintne inter ſe, ut Quantitas Rationalis ad Quantitatem Rationalem. Quando enim hoc contingit, ſurdæ commenſurabiles pronunciantur; aliter incommenſurabiles. Hoc modo, $\sqrt{75xx}$ & $\sqrt{27xx}$, diviſione factâ per $\sqrt{3}$, maximum earum Diviſorem communem, evadunt $\sqrt{25xx}$ & $\sqrt{9xx}$, ſive $5x$ & $3x$: & ergo ſurdæ propoſitæ ſunt inter ſe, ut numerus 5 ad 3, ac propterea *Commenſurabiles*. Sic $\sqrt{q^4+qqbb}$ & $\sqrt{qqbb+b^4}$, diviſæ per $\sqrt{qq+bb}$; dant $\sqrt{qq}$ & $\sqrt{bb}$, ſive q & b, adeo ut prima ſit $= q\sqrt{qq+bb}$, & ſecunda $= b\sqrt{qq+bb}$, & idcirco ad invicem, ut Rationalis q ad Rationalem b. Haud aliter, $\sqrt{\frac{eexx}{bb}+\frac{4nqxx}{bb}}$ & $\sqrt{\frac{bbeenn}{qqxx}+\frac{4bn^3}{qxx}}$ diviſæ per $\sqrt{ee+4nq}$, evadunt $\frac{x}{b}$, & $\frac{bn}{qx}$. Necnon $\sqrt{\frac{9a^2b-12abx+4bx^2}{bb}}$ $\sqrt{9a^2x+12ax^2-4x^3}$ & $\sqrt{\frac{16bc^2-24bcx+9bx^2}{dd}}$ $\sqrt{16c^2x+24cx^2-9x^3}$, diviſæ per $\sqrt{b-x}$, reddunt $\sqrt{\frac{9a^2-12ax+4x^2}{bb}}$, & $\sqrt{\frac{16c^2-24cx+9x^2}{dd}}$, vel $\frac{3a-2x}{b}$, & $\frac{4c-3x}{d}$.

Adeò ut *priores* duæ, ſint $\frac{x}{b}\sqrt{ee+\frac{4nq}{\alpha}}$, $\frac{bn}{qx}\sqrt{ee+4qn}$ & propterea, ut $\frac{x}{b}$ ad $\frac{bn}{qx}$, ſive ut qxx ad bbn: duæ *Poſteriores* ſunt, $\frac{3a-2x}{b}\sqrt{b-x}$, & $\frac{4c-3x}{d}\sqrt{b-x}$; quæ eandem rationem habent, quam $\frac{3a-2x}{b}$ ad $\frac{4c-3x}{d}$, ſive $3ad-2dx$ ad $4bc-3bx$.

Ex hinc, via per quam facilis patebit, ad Quantitatum Surdarum, *Additionem & Subtractionem* perficiendam. Factâ enim earum, ad terminos ſimpliciſſimos, reductione; ſi *Commenſurabiles* reperiantur, nil aliud opus eſt, quam ut *Quantitates, extra ſignum Radicale, Poſita*, ſibi mutuò *Addantur*, vel *Subtrahantur*, & *ſumma* vel *differentia*, in Quantitatem *ſub ſigno* incluſam, ducatur.

Ut $\sqrt{75aa}$ & $\sqrt{27aa}$, ſive $5a\sqrt{3}$ & $3a\sqrt{3}$; ſi *Addantur*, componunt $8a\sqrt{3}$, ſi vero *Subtrahatur* minor è majori, relinquitur $2a\sqrt{3}$. Sic $\sqrt{a^4+aabb}$ & $\sqrt{aabb+b^4}$, ſive $a\sqrt{aa+bb}$, & $b\sqrt{aa+bb}$, *Addendo* & *Subtrahendo*, faciunt *ſummam* $a+b\sqrt{aa+bb}$ & *differentiam*, $a-b\sqrt{aa+bb}$, vel $b-a\sqrt{aa+bb}$, prout a vel b major eſt.

Similiter $\sqrt{\frac{4eedd-4eezz}{ee}}$ (hoc eſt, per reductionem, $\frac{2e}{e}$, $\sqrt{dd-zz}$) & $\sqrt{dd-zz}$, vel Ix $\sqrt{dd-zz}$; dant *ſummam* $\frac{2e+e}{e}\sqrt{dd-zz}$, & *differentiam* $\frac{2e-e}{e}\sqrt{dd-zz}$ vel $\frac{e-2e}{e}\sqrt{dd-zz}$.

Ubi

Ubi vero, Quantitates Surdæ, *Incommenſurabiles* exiſtunt, ſit earum *Additio* & *Subtractio*, ope ſignorum + & —, per quæ copulari debent. Ut ſumma duarum $e+z$ & $\sqrt{ee-zz}$, eſt $e+z+\sqrt{ee-zz}$. *Differentia* (ſi poſterior à priore ſubducatur) $e+z-\sqrt{ee-zz}$; nec operæ *pretium* erit, plura hujus generis exempla, in medium proferre, eodem enim modo in cæteris caſibus operandum eſt, explorata prius Quantitatum propoſitarum Incommenſurabilitate.

Inſtituitur *Multiplicatio*, Quantitatum Surdarum (Commenſurabilium) ducendo partes, quæ *extra ſignum Radicale* ſunt, in ſe mutuò; & productum, in id, quod *ſub ſigno Radicali* continetur. Sic multiplicando $\sqrt{20xx}$ per $\sqrt{125xx}$, vel $2x\sqrt{5}$, per $5x\sqrt{5}$, fit $10xx \times 5 = 50xx$; & $\sqrt{16eed+16eec}$ per $\sqrt{81eed+81eec}$, ſive $4e\sqrt{d+c}$ per $9e\sqrt{d+c}$ pritur $36ed+c = 36eed+36eec$; & $\sqrt{q^4+qqbb}$ per $\sqrt{qqbb+b^4}$, ſive $q\sqrt{qq+bb}$ per $b\sqrt{qq+bb}$, fit $bq \times qq+bb = bq^3+qb^3$. Haud aliter, ſi proponatur expreſſio $\sqrt{z^4+6z^3+21zz+72z+108}$, multiplicanda per expreſſionem $\sqrt{z^4-10z^3+37zz-120z+300}$ id eſt $z+3\sqrt{zz+12}$, per $5-z\sqrt{zz+12}$: tum ductis $z+3$ & $5-z$ in ſe, fit $15+2z-zz$, quæ per $zz+12$ multiplicata, dat $180+24z+3zz+2z^3-z^4$, pro producto quæſito.

Si vero Surdæ datæ, *incommenſurabiles* ſint, tantum opus eſt, quantitates *ſub ſignis Radicalibus comprehenſas*, in ſe mutuò ducere; & producto commune ſignum radicale præfigere.

Sic $\sqrt{5}$ per $\sqrt{7}$, eſt $\sqrt{35}$; $\sqrt[3]{4}$ per $\sqrt[3]{9}$ eſt $\sqrt[3]{36}$; $\sqrt{ax}$ per $\sqrt{dz}$; eſt $\sqrt{adxz}$; $\sqrt{cc+dd}$ per $\sqrt{cc-dd}$, dat $\sqrt{c^4-d^4}$: Haud ſecus, $e+\sqrt{bx}$, in ſeipſam ducta, fecit $ee+bx+2e\sqrt{bx}$: hoc eſt, ſummam Quadratorum, *Rationalem*, & pro duplo Rectangulo, *Irrationalem* $2e\sqrt{bx}$ vel $\sqrt{4eebx}$. Eodem modo ſi $\sqrt{pp+xx}+\sqrt{pp-xx}$ multiplicanda eſſet per $\sqrt{pp+xx}-\sqrt{pp-xx}$; tum ductis duobus primis membris in ſe, fit $pp+xx$, & etiam duobus ultimis (habita ratione ſignorum contrariorum) in ſe, fit $-pp+xx$; & multiplicando utrinq; prima & ultima in ſe, hoc eſt $\sqrt{pp+xx}$ in $\sqrt{pp-xx}$, $\sqrt{pp+xx}$ in $-\sqrt{pp-xx}$, producta, utpote ſignis contrariis notata, ſe mutuo deſtruunt adeo ut relinquantur tantum $pp+xx$ & $-pp+xx$, ſibi invicem pro producto, quod idcirco ſolummodo ipſi $2xx$ æquale eſt.

Præterea, Quantitatum Surdarum *Diviſio*, ſi *commenſurabiles* ſunt, perficitur dividendo Quantitates, *extrà* ſignum radicale poſitas; quod enim ex diviſione oritur, *Quotiens* eſt.

Sic ad dividendum $\sqrt{e^8+e^6dd}$ per $\sqrt{e^4a^4+e^2a^4a^2}$, ſive $e^2\sqrt{e^4+eedd}$, per $a^2\sqrt{e^4+e^2a^2}$, fit $\frac{e^2}{a^2}$; & $\sqrt[3]{abc^3}$ diviſa per $\sqrt[3]{ab}$, hoc eſt $c\sqrt[3]{ab}$ per $\sqrt[3]{ab}$, dat c. Et modo conſimili in cæteris agendum eſt.

Si vero *incommenſurabiles* ſunt, tum ſolummodo dividendæ ſunt quantitates, *ſub ſignis Radicalibus comprehenſæ*, & ei quod reſultet *commune* ſignum radicale præfigendum eſt.

Ut

Ut $\sqrt[3]{a^3b-ab^3}$ divisa per $\sqrt[3]{aa-bb}$, facit $\sqrt[3]{ab}$; nam a^3b-ab^3 per $aa-bb$ divisa, ponit in Quoto ipsam ab.

Et quidem si signa Radicalia *diversa* fuerint, tunc quantitates datæ, ad *commune signum* radicale reducendæ erunt: Quod etiam in omni alia operatione, cum Surdis Heterogeneis observandum est.

Sic ad dividendum x^3+xcc per $\sqrt{x^4+xxcc}$ reducta expressione x^3+xc^3, ad Surdam ordinis Quadratici, evadit $\sqrt{x^6+2x^4c^2+x^2c^4}$; & hâc per priorem divisâ, oritur $\sqrt{xx+cc}$. Porrò si $x^3+xcc+xc\sqrt{xx+cc}$ dividenda sit per $x\sqrt{xx+cc}$; tunc Primo membro x^3+xcc, vel $\sqrt{x^6+2x^4c^2+x^2c^4}$ diviso per $x\sqrt{xx+cc}$, reddit in Quotiente $\sqrt{xx+cc}$; & secundo $xc\sqrt{xx+cc}$, per eandem $x\sqrt{xx+cc}$ etiam diviso, dat c; unde Quotiens quæsitus erit $\sqrt{xx+cc}+c$.

In Surdis vero *Universalibus*, ubi expressiones datæ, adeo sunt perplexæ & intricatæ, ejusve naturæ, ut divisio accuratè & sine Residuo nullo modo procedet, tunc in modum Fractionis Quotiens designari potest, collocatâ nimirùm Quantitate Dividente, tanquam Denominatore, sub alterâ, & *præfixo signo Radicali Universali*. Hoc modo, si exprimendus esset Quotiens, hujus Divisionis, nempe, $\sqrt{\sqrt{abb+bcd}}$, per $\sqrt{\sqrt{ac-dd}}$ scribere possumus, $\sqrt{\frac{\sqrt{abb+bcd}}{\sqrt{ac-dd}}}$; & $\sqrt{\frac{\sqrt{5+\sqrt{8}}}{\sqrt{7-\sqrt{2}}}}$, pro quotiente, orituro ex divisione $\sqrt[3]{\sqrt{5+\sqrt{8}}}$ per $\sqrt[3]{\sqrt{7-\sqrt{2}}}$. Et in horum Exemplorum similitudinem, quæ hujusque breviter, & alio Festinantes, proposuimus, alia sibi fingere potest, Tyro Algebraicus:

deo ut opus non ſit, de hiſce, plura diſſerere.

Reſtat ſolummodo, ut Praxin utiliſſimam ſimul ac facillimam, paucis exponam, ad multinomia, è quantitatibus Irrationalibus compoſitas pertinentem. Nempe qua ratione, Quantitates Rationales produci poſſunt, per aptas (& quoties res poſtulat) repetitas, horum Polynomiorum multiplicationes.

Et ut hujus operationis, Fundamentum verum ac genuinum, planius innoteſcat, Quantitates Surdas, non per ſua ſigna Radicalia ſed per Indices (ſive exponentes) hoc loco deſignabimus.

Praxis enim, hoc modo, multo facilior ac ſimplicior evadit, & producta oriunda, in Caſibus, innumeris, minori opere previdentur ac formantur.

Ex. gr. Proponatur $\sqrt{a}+\sqrt{b}$, ſive $a^{\frac{1}{2}}+b^{\frac{1}{2}}$; ex cujus multiplicatione per aliam quandam quantitatem, oriri debet *quantitas rationalis*. Patet primò, quod quantitas huic fini congrua, ex tot *Nominibus*, conſtare debet, quot conſtat ipſa propoſita: hoc eſt, Binomium eſſe debet. Deinde ſi $a^{\frac{1}{2}}+b^{\frac{1}{2}}$ ducatur in ſeipſam, id eſt, ſi multiplicemus $a^{\frac{1}{2}}+b^{\frac{1}{2}}$ per $a^{\frac{1}{2}}+b^{\frac{1}{2}}$, productum, duo membra rationalia, & Tertium irrationale ſive Surdum, comprehendet, *viz.* a, b, & $2a^{\frac{1}{2}}b^{\frac{1}{2}}$ ſive $2\sqrt{a}\times\sqrt{b}$, hoc eſt $2\sqrt{ab}$. Quapropter $a^{\frac{1}{2}}+b^{\frac{1}{2}}$ non eſt Binomium illud conveniens, ex cujus multiplicatione in Surdam propoſitam, procreabitur Quantitas rationalis.

Sumatur ergo *Reſiduum* correſpondens, nempe $a^{\frac{1}{2}}-b^{\frac{1}{2}}$ (quod ipſum etiam, alſo ſenſu Binomium eſt,

eſt, etſi propter ſignum negativum, a *Binomio proprie ſic dicto*, diſtinguatur) Tum multiplicando $a^{\frac{1}{2}}+b^{\frac{1}{2}}$ per $a^{\frac{1}{2}}-b^{\frac{1}{2}}$, ſit productum rationale $a-b$; termini enim in quibus $a^{\frac{1}{2}}$ & $b^{\frac{1}{2}}$ in ſe ducti reperiuntur, propter ſigna contraria, ſe mutuo deſtruunt.

Mult. $a^{\frac{1}{2}}+b^{\frac{1}{2}}$
Per $a^{\frac{1}{2}}-b^{\frac{1}{2}}$
Prod. $a+a^{\frac{1}{2}}b^{\frac{1}{2}}-a^{\frac{1}{2}}b^{\frac{1}{2}}-b$, ſive $a-b$.

Sic res ſe habet, in Binomiis, ex *puris radicibus Quadraticis*, conſtantibus. In aliis, Plures adhuc multiplicationes requiruntur.

Sit Binomium, ex Radicibus Biquadraticis; *ex. gr.* $\sqrt[4]{a}+\sqrt[4]{b}$, vel $a^{\frac{1}{4}}+b^{\frac{1}{4}}$; tunc ut prius.

Mult. $a^{\frac{1}{4}}+b^{\frac{1}{4}}$
Per $a^{\frac{1}{4}}-b^{\frac{1}{4}}$
Prod. $a^{\frac{1}{2}}-b^{\frac{1}{2}}$, ſive $\sqrt{a}-\sqrt{b}$.

Jam Mult. $a^{\frac{1}{2}}-b^{\frac{1}{2}}$
Per $a^{\frac{1}{2}}+b^{\frac{1}{2}}$
Prod. $a-b$, ut antea.

Proponatur Binomium ex Radicibus *Octanii*, ſive Biquadraticarum --- Quadraticis, conſtans.

Mult. $a^{\frac{1}{8}} + b^{\frac{1}{8}}$

Per $a^{\frac{1}{8}} - b^{\frac{1}{8}}$

Prod. $a^{\frac{1}{4}} - b^{\frac{1}{4}}$, vel $\sqrt[4]{a} - \sqrt[4]{b}$.

Iterum, Mult. $a^{\frac{1}{4}} - b^{\frac{1}{4}}$

Per $a^{\frac{1}{4}} + b^{\frac{1}{4}}$

Prod. $a^{\frac{1}{2}} - b^{\frac{1}{2}}$, vel $\sqrt{a} - \sqrt{b}$.

Ultimo, Mult. $a^{\frac{1}{2}} - b^{\frac{1}{2}}$

Per $a^{\frac{1}{2}} + b^{\frac{1}{2}}$

Prod. $a - b$

Et ſic patet progreſſus in Infinitum, cum Radices ad *ordinem Quadraticum* pertinent. In *aliis*, res non æquè ſuccedet, cum oriantur producta, partim Surda, partim rationalia; idq; ex exponentium natura ac genio. Ut ſi *ex. gr.*

Mult. $a^{\frac{1}{3}} + b^{\frac{1}{3}}$, hoc eſt $\sqrt[3]{a} + \sqrt[3]{b}$,

Per $a^{\frac{1}{3}} - b^{\frac{1}{3}}$, hoc eſt $\sqrt[3]{a} - \sqrt[3]{b}$,

Prod. $a^{\frac{2}{3}} - b^{\frac{2}{3}}$, ſive $\sqrt[3]{aa} - \sqrt[3]{bb}$.

Si productum hoc poſterius, *viz.* $a^{\frac{2}{3}} - b^{\frac{2}{3}}$ ducas in $a^{\frac{2}{3}} + b^{\frac{2}{3}}$, habebis tantum $a^{\frac{4}{3}} - b^{\frac{4}{3}}$, & ſi eodem modo, ad Infinitum uſque progrediaris, productum rationale, nunquam adipiſceris. Si vero $a^{\frac{1}{3}} - b^{\frac{2}{3}}$ ducatur in $a^{\frac{1}{3}} + b^{\frac{1}{3}}$, orietur mixtum

tum hoc $a - b + a^{\frac{2}{3}} b^{\frac{1}{3}} - a^{\frac{1}{3}} b^{\frac{2}{3}}$, id eſt, $a - b + \sqrt[3]{aa}\sqrt[3]{b} - \sqrt[3]{a}\sqrt[3]{bb}$.

In *Trinomiis*, *Quadrinomiiſve* ex puris *Radicibus Quadraticis* conflatis, multiplicatio per Trinomia, vel Quadrinomia convenientia, inſtituenda eſt; qua ratione *Quadrinomia* ad *Trinomia*, & *Trinomia* ad *Binomia* deprimentur & inde per multiplicationem cum propriis *Reſiduis* ad *Quantitates uſque rationales* perducentur.

Sit *ex. gr.* Trinomium $\sqrt{a} + \sqrt{b} + \sqrt{c}$, vel $a^{\frac{1}{2}} + b^{\frac{1}{2}} + c^{\frac{1}{2}}$, ſi fieret hujus multiplicatio, cum Trinomio ex iiſdem nominibus conſtante, & eodem ſigno affectis, liquet, quod præter terminos rationales $a+b+c$, etiam ſex alii irrationales in producto remanerent; adeo ut ex hac operatione nihil omnino proficeremus. Si vero mutato ſigno unius exnominibus, multiplicatore $\sqrt{a}+\sqrt{b}-\sqrt{c}$ uteremur, tum tot membra in producto ſe mutuo tollerent, ut reliquorum ſumma, non ultra Binomium aſcenderet.

Mult. $a^{\frac{1}{2}} + b^{\frac{1}{2}} + c^{\frac{1}{2}}$

Per $a^{\frac{1}{2}} + b^{\frac{1}{2}} - c^{\frac{1}{2}}$

Prod. $a + b - c + 2\,a^{\frac{1}{2}}\,b^{\frac{1}{2}}$, ſive ponendo $a + b - c = d$, erit $d + 2\sqrt{ab}$. Aſſumpto tunc *Reſiduo* $d - 2\sqrt{ab}$, vel $2\sqrt{ab} - d$.

Mult. $d + 2\,a^{\frac{1}{2}}\,b^{\frac{1}{2}}$

Per $d - 2\,a^{\frac{1}{2}}\,b^{\frac{1}{2}}$

Prod. Rat. $d^2 - 4ab$, vel $4ab - d$ ſi $4ab > d$.

Jam

Jam ut in Trinomio, unius ſolum nominis, ſic in Quadrinomio, *duorum nominum ſigna mutanda ſunt*, ut multiplicationis opo ad Trinomium deprimatur.

Sit $a^{\frac{1}{2}}+b^{\frac{1}{2}}+c^{\frac{1}{2}}+d^{\frac{1}{2}}$

Mult. per $a^{\frac{1}{2}}+b^{\frac{1}{2}}-c^{\frac{1}{2}}-d^{\frac{1}{2}}$, & rejectis contrariis.

Sit Prod. $a+b-c-d+2\sqrt{ab}-2\sqrt{dc}$, & loco $a+b-c-d$ ſcribendo f, habemus Trinomium $f+2\sqrt{ab}-\sqrt{dc}$. Et operando ut prius.

Mult. $f+2a^{\frac{1}{2}}\ b^{\frac{1}{2}}-2d^{\frac{1}{2}}\ c^{\frac{1}{2}}$

Per $f+2a^{\frac{1}{2}}\ b^{\frac{1}{2}}+2d^{\frac{1}{2}}\ c^{\frac{1}{2}}$, & rejectis rejiciendis.

Fit Prod $f^2+4ab-4dc+4fa^{\frac{1}{2}}\ b^{\frac{1}{2}}$; & ponendo $f^2-4ab-4dc=n^2$, fit Prod. $n^2+4f\ a^{\frac{1}{2}}\ b^{\frac{1}{2}}$.

Ultimo, Mult. $n^2+4fa^{\frac{1}{2}}\ b^{\frac{1}{2}}$

Per $n^2-4fa^{\frac{1}{2}}\ b^{\frac{1}{2}}$

Oritur Prod. n^4-16f^2ab, cujus nulla pars irrationalis eſt.

Hujus Praxeos uſus eſt, in *Quantitatum Surdarum Diviſionibus*, nempe cum Diviſor ipſe, Binomium vel Trinomium, *&c.* exiſtit. Reducto enim *Diviſore* ad ſimplicem quantitatem rationalem, & facta multiplicatione Quantitatis Dividendæ, per eaſdem omnes quantitates quibus uſi fuimus in quærendo diviſore rationali; conſequemur hoc modo novum diviſorem, novamque Dividendam,

videndam, quæ eandem inter se proportionem habebunt, ac priores. Ut si requiritur quotiens ex divisione $\sqrt{8}$ per $\sqrt{3}+\sqrt{2}+1$; Multiplico divisorem per $\sqrt{3}+\sqrt{2}-1$, & invenio $\sqrt{24}+4$, quæ ducta in $\sqrt{24}-4$ (proprium ejus *Residuum*) dat rationalem divisorem 8. Et quoniam *Divisor* $\sqrt{3}+\sqrt{2}+1$, primo multiplicatus fuit per $\sqrt{3}+\sqrt{2}-1$, & productum per $\sqrt{24}-4$, ergo & quantitas *Dividenda* multiplicari debet, per $\sqrt{3}+\sqrt{2}-1$. & productum per $\sqrt{24}-4$, sic ut habeamus $8+\sqrt{128}-192$ pro nova Dividendâ, quâ per 8 divisâ, emergit quotiens $1+\sqrt{2}-\sqrt{3}$.

Et Pauca hæc, de Surdis dicta, sufficiant.

Restat tantum ut subnectamus Regulam quandam egregiam ac facilem, pro Radicum extractione, e Binomiis ex Integris & Radicalibus *Quadraticis* constantibus; quam nobis suppeditavit *vir Præclarissimus juxta ac Honoratissimus* D. *Isaacus Newtonus*, in Algebrâ suâ nuper editâ (Prole, Tanto Parente undequaque dignâ) Sit Binomii alicujus Pars major A, minor B: erit tum $\frac{A+\sqrt{AA-BB}}{2}$ Quadratum majoris partis Radicis, & $\frac{A-\sqrt{AA-BB}}{2}$ Quadratum partis minoris, quæ majori connectenda est cum signo ipsius B; adeoque erunt $\sqrt{\frac{A+\sqrt{AA-BB}}{2}}$ & $\sqrt{\frac{A-\sqrt{AA-BB}}{2}}$ ipsæ partes Radicem componentes. *Ex. gr.* proponatur Binomium $14+\sqrt{180}$, ubi $A=14$, $B=\sqrt{180}$, est enim $14\times 14\ (=196) > 180$. Jam $AA-BB=16$, $\therefore \sqrt{AA-BB}=4$,

$\therefore$

$\therefore \frac{A+\sqrt{AA-BB}}{2} = \frac{14+4}{2} = 9$. Similiter $\frac{A-\sqrt{AA-BB}}{2} = \frac{14-4}{2} = 5$. Adeo ut Quadrata partium Radices ſint 9 & 5, unde Radix ipſa $=3+\sqrt{5}$.

Sit Binomium $\frac{2a^4+x^4}{x^2} + \frac{2a^2\sqrt{a^4+x^2}}{x}$, cujus Radix Requiritur. Hic $\frac{2a^4+x^4}{x^2}$ ſive $\frac{2a^4}{x^2}+x^2 = A$, & $\frac{2a^2}{x}\sqrt{\frac{a^4}{x^2}+x^2} = B$, & $AA-BB = x^4$, $\therefore \sqrt{AA-BB} = x^2$. Ergo erit $\frac{A+\sqrt{AA-BB}}{2} = \frac{a^4}{x^2}+x^2$, & $\frac{A-\sqrt{AA-BB}}{2} = \frac{a^4}{x^2}$. Unde ipſa Radix $= \frac{a^2}{x} + \sqrt{\frac{a^4}{x^2}+xx}$.

Rurſus proponatur $aa+5ax-2a\sqrt{ax+4xx}$ (quod unum eſt de exemplis ipſius *viri Clariſſimi*) Ponendo $aa+5ax$ loco A, & $2a\sqrt{ax+4xx}$ pro B, erit $AA = a^4+10a^3x+25a^2x^2$, $BB = 4a^3x+16a^2x^2$, $AA-BB = a^4+6a^3x+9a^2x^2$, $\sqrt{AA-BB} = aa+3ax$; unde $\frac{A+\sqrt{AA-BB}}{2} = \frac{aa+5ax+aa+3ax}{2} = aa+4ax$ = Quadrato majoris

joris partis Radicis, & $\frac{A-\sqrt{AA-BB}}{2}=$

$\frac{aa+5ax-aa-3ax}{2}=ax$. Adeoque Radix integra $=$

$\sqrt{aa+4ax}-\sqrt{ax}$, pars minor enim majori connectanda est, cum signo ipsius B, quod hoc in loco negativum est.

Jam veritas hujus Regulæ hinc *demonstratur*, quod cum Radicis Pars major, ex ejus præscripto, fit $\sqrt{\frac{A+\sqrt{AA-BB}}{2}}$ & minor $\sqrt{\frac{A-\sqrt{AA-BB}}{2}}$, adeoque earum summa, integræ Radici æqualis; si aggregatum hoc in se multiplicetur, Productum ipsi A+B æquale esset debet. Et ex harum Surdarum constitutione, manifestum est nullum aliud Productum oriri posse. Quadratum enim membri majoris est $\frac{A+\sqrt{AA-BB}}{2}$, & minoris $\frac{A-\sqrt{AA-BB}}{2}$, quorum summa $=$ A, duplum vero Rectangulum, sive $\sqrt{\frac{A+\sqrt{AA-BB}}{2}}$ in $\sqrt{\frac{A-\sqrt{AA-BB}}{2}}$ bis sumptum, dabit tantummodo B. Nam per Regulas communes, si accipiatur Radix Producti partium, sub signo Universali comprehensarum, erit illa æqualis producto ipsarum Radicum universalium, & ergo hæc bis sumpta dabit duplum rectangulum. Sed $\frac{A+\sqrt{AA-BB}}{2}$ in $\frac{A-\sqrt{AA-BB}}{2}=$

$\frac{AA+A\sqrt{AA-BB}-A\sqrt{AA-BB}-AA-BB}{4}$

$= \frac{AA - \sqrt{AA - BB}}{4} = \frac{BB}{4}$, cujus Radix $= \frac{B}{2}$, & ejus dupla $= B$. Patet ergo propositum.

At vero *Investigatio* hujus Regulæ hoc modo procedere potest. Sit Binomium $p+q$, cujus nomen majus p. In *Quadrato*, summa Quadratorum duarum partium radicis, reliquo membro, major semper erit. Termini enim ipsi (sepositis numeris) sunt, continuè proportionales. Et in Tribus continuè proportionalibus, ut ee, ne, nn, summa extremorum $ee+nn$ termino intermedio $2ne$, semper major est. Ac propterea nomen p, tanquam summa Quadratorum duarum partium Radicis, concipi debet. Ponatur una pars Radicis $= x$, ergo $\sqrt{p-xx} =$ alteri, ergo Quadratum hujus expressionis $x+\sqrt{p-xx}$, nempe $p+2x\sqrt{p-xx} = p+q$, vel $2x\sqrt{p-xx} = q$. Unde $4pxx - 4x^4 = qq$, sive $4x^4 - 4pxx = -qq$, & ponendo $xx = y$, erit $yy - py = -qq$, adeoque $yy - py + \frac{1}{4}pp = pp - qq$, & $y - \frac{1}{2}p = \sqrt[4]{\frac{pp-qq}{4}} = \frac{\sqrt{pp-qq}}{2}$, ergo $y = \frac{p+\sqrt{pp-qq}}{2}$. Et cum sit $y = xx =$ Quadrato unius partis Radicis; & $p - xx =$ Quadrato alterius, quod ergo æquatur $p - \frac{p+\sqrt{pp-qq}}{2} = \frac{p-\sqrt{pp-qq}}{2}$; hinc $\frac{p+\sqrt{pp-qq}}{2}$ est Quadratum majoris, & $\frac{p-\sqrt{pp-qq}}{2}$ Quadratum minoris partis, Radicis, hoc est $\frac{A+\sqrt{AA-BB}}{2}$, &

$\frac{A - \sqrt{AA-BB}}{2}$, ſcribendo A & B pro p & q, prout nos edocet Regula ſuperiùs laudata.

Quod idem, & ex alio Principio, ſic colligi poteſt. Sit Quadratum aliquod $n^2+2nm+m^2$ ex radice $n+m$ genitum. Accipiatur *ſummæ* Quadratum ex partibus radicis, pro *primo* membro, & duplum Rectangulum pro *ſecundo*; adeo ut ſeorſim ſumpta ſint n^2+m^2, & $2nm$. Si de *primi* membri Quadrato, auferatur Quadratum *ſecundi*, Radix Quadrata Reſidui, erit Differentia Quadratorum partium radicis. Hoc eſt $n^4+2n^2m^2+m^4-4n^2m^2 = n^4-2n^2m^2+m^4$, cujus radix $= n^2-m^2$. Sed $\frac{n^2+m^2}{2} + \frac{n^2-m^2}{2} = n^2$, & $\frac{n^2+m^2}{2} - \frac{n^2+m^2}{2} = m2$, nempe Dimidium *Summæ* & *Differentiæ*, duarum quarumvis quantitatum, ſi ſibi invicem Addantur & Auferantur, conficiunt ipſas Quantitates. Ponendo

Jam $n^2+m^2 = d^2$, erit propterea

$\frac{d^2 + \sqrt{n^4-2n^2m^2+m^4}}{2} = n^2$, &

$\frac{d2 - \sqrt{n4-2n2m^2+m4}}{2} = m2$, vel æqualium ſubſtitutione, fit $\frac{d^2 + \sqrt{d^2-4n^2m^2}}{2} = n^2$, &

$\frac{d^2 - \sqrt{d^2-4n^2m^2}}{2} = m^2$, quæ expreſſiones prorſus eædem ſunt cum *prioribus*.

CAP.

CAP. II.

De Modo Separandi, & Exprimendi Quantitates Incognitas, quando proponuntur Æquationes, quæ plures comprehendunt.

UT Opus hoc felicitèr abſolvat *Tyro Algebraicus*, duo ſunt imprimis, quæ in ipſo limine, acri Judicio perpendere debet. Nempe, *Primo, An Æquationes propoſitæ, ſine realiter diſtinctæ, & a ſe invicem nullo modo dependeant? Secundo, An ſint Plures, vel Pauciores, vel Æquales Numero, cum Quantitatibus Incognitis*, quæ in ipſis continentur. Quando enim *tot* habentur æquationes, *quot* incognitæ, & *viciſſim*; tunc omnes ad unam ſemper reduci poſſunt, in quâ *unica* ſolum incognita ſupererit.

Hoc eſt, omnes Incognitæ, per *commodas ſubſtitutiones* exterminari poſſunt, unâ tantum exceptâ, cujus valor in *puris cognitis* habebitur; & inde per *datas relationes*, cæterarum incognitarum ad hanc unam, ipſæ quoque nullo negotio detegentur. Si *plures ſint æquationes*, quam incognitæ, tunc ob ſupervacaneum æquationum numerum, neceſſe eſt *varias oriri limitationes* quantitatum inter ſe, & (ex ſuppoſitionibus quibuſdam repugnantibus) ſæpenumerò contradictorium Aliquid,

Aliquid, vel Impoſſibile. Si vero *plures ſint incognitæ*, quam æquationes, hoc non obſtante, incognitarum una vel plures, ſemper tolli poſſunt, & hoc modo inveniri equatio, qua' reliquarum Relatio mutua determinetur.

Finitâ enim operatione, una vel plures incognitæ manebunt, quoniam, unius æquationes ope, una ſolum incognita poteſt auferri. Ac propterea tot nunquam auferri poterunt, quot in æquationibus comprehenduntur; cum ex Hypotheſi, earum numerus, æquationum propoſitarum numerum excedat. Sed quo major ſit iſte exceſſus, eò plures incognitarum ultimò manebunt. Numeruſq; earum ultimò ſic manentium, æqualis erit aggregato *unitatis* & Differentiæ prædictæ. Hoc eſt ſi incognitæ ſint æquationibus unâ plures, duæ manebunt, ſi duabus, Tres, & ſic deinde.

Sed exempla jam proferemus, quibus Generalia hæc melius Intelligantur.

Et ut a Facilioribus incipiamus, primo loco tractabimus æquationes, in quibus *Quantitates incognitæ, nec in ſe invicem multiplicantur, nec ad ullam poteſtatem aſcendunt.* Quæ iterum in duas diverſas claſſes diſpoſci poſſunt. Vel enim, ita ordinatur propoſitæ æquationes ut ſingulæ incognitæ, in ſingulis æquationibus reperiantur; vel e contrario, ut quædam ex ipſis in quibuſdam tantum æquationibus, & non in aliis, locum habeant.

In *Primo Caſu*, Operatio Generaliter hoc modo inſtitui poteſt.

Cujuſvis Incognitæ (quam *Primam* nominabimus) valores duo, ſibi mutuò æquentur, & inde reperiatur expreſſio *ſecundæ* alicujus Incognitæ, in Terminis partim *cognitis*, & partim reliquas

incognitas includentibus. Tum repetitâ operatione, inveniatur altera consimilis relatio, hujus *secundæ* Incognitæ, ad terminos quosdam ex cognitis & incognitis Quantitatibus compositos. Unde factâ æquatione, inter duos *secundæ* hujus Incognitæ repertos valores; dabitur expressio *Tertiæ* cujusdam Incognitæ, in Terminis, ex cognitis & (Paucioribus quam prius) Incognitis, constantibus. Et per binos hujus *Tertiæ* valores, simili prorsus modo exprimetur *Quartæ* Incognita, in Quantitatibus datis, & Paucioribus adhuc Incognitis. Et sic deinceps, quantuscunq; sit æquationum propositarum numerus.

Ultimo enim orietur æquatio, *unicam tantum incognitam*, (cum *datis* quantitatibus) complexa; si modo tot sint Incognitæ, quot æquationes, & *vicissim*. Et Hæc Argumentandi methodus, *Generalis*, *Certa*, ac *Perspicua* est; concesso interim, in casibus quibusdam particularibus, rem minori forsan circuitu perfici posse. Sed *compendia* hic tradere, præter institutum est, nec Tyronibus adeo necessarium; cum *usus* & *exercitium*, ingegenio sagaci ac sedulo, omnia hæc *inferioris notæ* auxilia abundè suppeditabunt, Regula quædam constans ac Uniformis ipsis potiùs monstranda foret, cujus ope Quantitates incognitas, in omnia casu, eruere & exprimere valerent.

Sint Æquationes, $\left.\begin{matrix} mx+ny=p \\ bx+cy=q \end{matrix}\right\}$

Ex

$$\text{Ex } \begin{matrix}1^{ma}\\2^{da}\end{matrix} \left\{\begin{matrix}\\\\\end{matrix}\right. \text{oritur} \left\{\begin{matrix} x = \frac{p - ny}{m} \\ x = \frac{q - cy}{b} \end{matrix}\right.$$

$\therefore$ Ex utriſq; $\frac{p-ny}{m} = \frac{q-cy}{b}$, ſive reducendo $y = \frac{bp-mq}{n-c}$. Unde cognoſcitur x, ope utriuſlibet æquationis, valorem ejus in y & terminis cognitis, exprimentis.

Notandum, in exemplis hiſce *Generalibus*, Coefficientes cæteraſq; quantitates (quæ ut cognitæ ſpectantur, & Magnitudinis prorſus indeterminatæ ſunt) *ad libitum tranſponi poſſe*, quod in exemplis *numericis* non fit. Finis enim unicus ad quem hîc reſpicimus, eſt ut quantitates incognitæ in terminis æquationum propoſitarum, verè & aptè explicentur.

$$\text{Sint Æq.} \quad \left\{\begin{matrix} dx = yz \\ x + y = \frac{z}{n} \\ px = y + z \end{matrix}\right.$$

$$\text{Ex } \begin{matrix}1^{ma}\\2^{da}\end{matrix} \left\{\begin{matrix}\\\\\end{matrix}\right. \text{oritur} \left\{\begin{matrix} x = \frac{yz}{d} \\ x = \frac{z}{p} - y \end{matrix}\right.$$

∴ Ex Utrisque $\frac{yz}{d} = \frac{z}{n} - y$, vel $dz - dny = nyz$, hoc est $y = \frac{dz}{nz+dn}$, adeoque habetur y per z & *cognitas* quantitates.

Ex $\left.\begin{matrix}2^{da}\\3^{tia}\end{matrix}\right\}$ est $\begin{cases} x = \frac{z}{n} - y \text{ (uti prius)} \\ x = \frac{y+z}{p} \end{cases}$

Unde conjunctim $\frac{z}{n} - y = \frac{y+z}{p}$ vel reducendo $y = \frac{pz-nz}{pn+n}$, quæ iterum exprimitur per z & cognitis. Ergo $\frac{dz}{nz+dn} = \frac{pz-nz}{pn+n}$, vel $pndz+ndz = pnzz-nnzz+pndz-dnzz$, hoc est abjecto utrinque $pndz$, & reliquis divisis *per* nz, obtinebitur expressio Quantitatis z ab omni Incognitâ libera.

Sint Æq. $\left.\begin{matrix} nx+my-pz=a \\ rx-qy+sz=b \\ gx+hy-lz=c \end{matrix}\right\}$

Ex $\left.\begin{matrix}1^{ma}\\2^{da}\end{matrix}\right\}$ Habemus $\left\{\begin{matrix} x = \frac{a+pz-my}{n} \\ x = \frac{b+qy-sz}{r} \end{matrix}\right\}$

∴ Ex

∴ Ex 1ma & 2da *conjunctim*, habemus, z, per y & cognitas, hoc eſt $ra + rpz - rmy = nb + nqy - nsz$, vel $z = \frac{nb - ra + nq + rm \times y}{rp + ns}$. Jam inquirenda eſt alia ſimilis expreſſio Quantitatis z. Ergo ex $\left.\begin{matrix}1^{ma}\\2^{da}\end{matrix}\right\}$ (hæ enim pro arbitrio ſumuntur, cum æquè liberum fuiſſet 2dam ac 3riam accepiſſe) oritur

$$\left.\begin{matrix} x = \frac{a + pz - my}{n} \text{ (ut prius)} \\ x = \frac{c + lz - hy}{g} \end{matrix}\right\} \text{∴ Ex } 1^{ma} \text{ \& } 3^{tia} \text{ conjunctim,}$$

habemus alium valorem ipſius z, per y & cognitas, nempe $ga + gpz - gmy = nc + lzn - hny$, vel $z = \frac{ga - nc + hn - gm \times y}{ln - gp}$. Quibus valoribus quantitatis z, inter ſe æquatis, habetur y in puris cognitis. Et regrediendo ad utramvis æquationem, qua exprimitur z per y & cognitas, innoteſcit etiam z, & per datas primò æquationes, x.

Sint Æq. $\left.\begin{matrix} x + nz + ny + nv = b \\ z + mx + my + mv = c \\ y + px + pz + pv = d \\ v + qx + qz + qy = f \end{matrix}\right\}$ .....

Ex $\left.\begin{matrix}1^{ma}\\2^{da}\end{matrix}\right\}$ $\left.\begin{matrix} x = b - nz - ny - nv \\ x = \frac{c - my - mv - z}{m} \end{matrix}\right\}$

∴ Ex 1ma & 2da conjunctim, habemus z expressam per y, v, & cognitas, id est

$$z = \frac{mb - c + \overline{n - mn} \times y + \overline{m - mn} \times v}{mn - 1}$$

Et ut alium obtineamus modum exprimendi z per y vel v, & cognitas quantitates,

Erit ex $\begin{matrix} 2^{da} \\ 3^{tia} \end{matrix}$ $\left\{ \begin{array}{l} x = \frac{c - my - mv - z}{m} \text{ (uti prius)} \\ x = \frac{d - pz - pv - y}{p} \end{array} \right\}$

∴ Ex 2da & 3^tia conjunctim oritur valor *secundus* quantitatis z, in y solâ & cognitis, hoc est $z = \frac{md - pc + \overline{pm - m} \times y}{mp - p}$ (etenim terminus mpv, propter signa contraria rejicitur (non minus tamen obtinuissemus expressionem ipsius z, in y, v, & cognitis si terminus iste adfuisset) ergo ex hisce duobus modis exprimendi quantitatem z, oritur æquatio, quâ *primo* exprimitur y, per v & terminos cognitos. Nempe hæc

$$\frac{mb - c + \overline{m - mn} \times y + \overline{m - mn} \times v}{mn - 1} = \frac{md - pc + \overline{pm - m} \times v}{mp - p}$$

Eodem

Eodem modo, æquatis duobus valoribus quantitatis x, ex æquationibus 3^tia & 4ta, desumptis obtinebitur *tertius* valor z, in v & cognitis; nempe $z = \frac{mf - qc + \overline{mq - m} \times v}{mq - q}$. Et ex duobus aliis valoribus ejusdem quantitatis x, ope æquationum 1ma & 4ta *Quarto* exprimitur z, in y, v, & cognitis, id est, $z = \frac{bq - f + \overline{q - nq} \times y + \overline{1 - nq} \times v.}{qn - q}$

Unde æquando duas hasce posteriores expressiones quantitatis z, erit $\frac{mf - qc + \overline{mq - m} \times v}{mq - q} = \frac{bq - f + \overline{q - nq} \times y + \overline{1 - nq} \times v}{qn - q}$: ex qua *secundo* exprimitur y, in v & cognitis. Adeò ut jam reperti sint, duo modi diversi, exprimendi incognitam y, in terminis v, & quantitatum datarum. Unde reperitur ipsa incognita v; & priorum æquationum ope, cæteræ quoque innotescunt.

Quando contingit ut singulæ incognitæ in singulis æquationibus non inveniantur, via tamen haud multum absimili earum expressiones elicere possumus: Inveniendo nempe diversos modos, quibus eadem incognita, in aliarum Incognitarum & Quantitatum datarum Terminis, explicari queat.

Ut ſi ex. gr. $\left.\begin{array}{l} px-y=a \\ ny-mz=b \\ z+nv=c \\ v-qx=d \end{array}\right\}$

Accipio ex Æq. 1ma & 4ta, duos valores quantitates x, *viz.* $x=\frac{a+y}{p}=\frac{v-d}{q}$, unde $y=\frac{pv-pd-qa}{q}$. Secundò; ex 2da & 4ta, habetur gemina expreſſio quantitatis z, *viz.* $=\frac{ny-b}{m}=c-nv$. Unde $y=\frac{mc-mnv+b}{n}$; adeo ut y bis jam exprimatur per v & cognitas. Ac propterea inventa eſt expreſſio quantitatis v; unde reliquæ ope æquationum datarum innoteſcunt.

Iterum ſint Æq. $\begin{array}{ll} x+y+z+v & =b \\ x+d & =y-d \\ x+d & =dz \\ x+d & =\frac{v}{d} \end{array}$

Ex $\left.\begin{array}{l} 1^{ma} \\ 2^{da} \end{array}\right\}$ Habemus $\left\{\begin{array}{l} x=b-y-z-v \\ x=y-2d \end{array}\right.$

∴ Ex utriſque, oritur $y=\frac{b+2d-z-v}{2}$.

Ex

Ex $\begin{matrix}3^{tia}\\4^{ta}\end{matrix}\Big\{$erit $dz-d\ (=x)=\frac{v}{d}-d$, adeoque $z=\frac{v}{dd}$, unde exterminatâ z, erit in æquatione priore, $y=\frac{b+2d-\frac{v}{dd}-v}{2}$. Quæratur alius consimilis valor quantitatis y.

Nempe ex $\begin{matrix}2^{da}\\4^{ta}\end{matrix}\Big\{$ est $\Big\}\ y-2d\ (=x)=\frac{v}{d}-d$, sive $y=\frac{v}{d}+d$. Et æquando valores hosce inventos quantitatis y, erit $\frac{b+2d-\frac{v}{dd}-v}{2}=\frac{v}{d}+d$.

Unde datur v, & per consequens cæteræ.

Si Æquationes datæ, *Potestates quasdam*, Incognitarum Quantitatum includant, ad earum expressiones eliciendas, frequenter per *Radicum adfectarum extractionem*, at interdum *sine eadem*, prodere potest *Algebræ studiosus*. Ex. gr.

Sint Æquationes $\left.\begin{matrix}x+y+\frac{yy}{x}=a\\xx+yy+\frac{y^4}{xx}=b\end{matrix}\right\}$

Erit ex $\begin{matrix}1^{ma}\\2^{da}\end{matrix}\left\{\begin{matrix}xx+yy=ax-xy\\x^4+xxyy+y^4=bxx\end{matrix}\right\}$ vel utrinq; addendo $xxyy$

$\left\{\begin{matrix}x^4+2xxyy+y^4=bxx+xxyy,\\\therefore \text{extr. rad. quad. } x^2+y^2=\sqrt{bxx+xxyy}\end{matrix}\right.$

unde cum habeatur gemina expressio quantitatis

$xx+$

$xx+yy$, erit $ax-xy=\sqrt{bxx+xxyy}$, vel $aaxx-2axxy+xxyy=bxx+xxyy$, $\therefore aaxx-2axxy=bxx$, vel $\frac{aa-b}{2a}=y$. Unde facile habetur expreſſio ipſius x, per ſolutionem æquationis *adfectæ*, ex. gr. $yy=ax-xy-xx$, in quâ y cognita eſt.

Sint Æq. $\left.\begin{array}{l} xx-yy=n \\ x^4-y^4=m \end{array}\right\}$

Quoniam nulla datur vel Potestas vel Productum, cujus duo diverſi valores æquationem poſſunt conſtituere: Ergo fiat Quadratum è prima expreſſione $xx-yy$, *viz.* $x^4-2xxyy+y^4$, ita enim ex utraque parte, invenietur x, ejuſdem dimenſionis. Adeò ut jam habeamus duas haſce æquationes,

$$\left.\begin{array}{rl} x^4-2xxyy+y^4 & =n^2 \\ x^4-y^4 & =m \end{array}\right\}$$

Unde ex $\left.\begin{array}{ll} 1^{ma} & y^4=n^2-x^4+2xxyy \\ 2^{da} & y^4=x^4-m \end{array}\right\}$

$\therefore$ Ex utriſque, $\frac{n^2+m}{2}=x^4-xxyy=\overline{xx-yy}$ *

$xx=\frac{xxx}{n}$, hoc eſt $xx=\frac{n^2+m}{2n}$. Et ope æquationis primæ exprimitur y, quoniam $xx-n=yy$.

Sint rurſus Æq. $\left.\begin{array}{l} xx+yy=b \\ xy+x+y=c \end{array}\right\}$ $\therefore xx+2xy+yy=b+2xy$, $\therefore xy=c-x-y$.

Po-

Ponatur $x+y=z$, erit $x=z-y$, & $xy=zy-yy$.
Tum facta ſubſtitutione,

Erit loco $\begin{matrix}1^{mæ}\\2^{dæ}\end{matrix}\begin{cases}zz=b+2zy-2yy\\zy-yy=c-z\end{cases}$

Et æquando duos valores Quantitatis $zy-yy$, habemus æquationem $\frac{zz-b}{2}=c-z$, vel $zz+2z=2c+b$. Cujus æquationis Reſolutione, exprimitur z in puris cognitis. Unde reſolvendo æquationem $zy-yy=c-z$ (in qua z cognita eſt) cognoſcitur etiam y. Et quoniam $z=y+x$, ergo etiam x in datis Quantitatibus exprimetur.

Nonnunquam verò Incognitarum expreſſiones, ad æquationes longe ſubliminores aſſurgunt.

Ut ſi $\begin{matrix}bbx-aby=aab+axy\\ \&\quad bxc+ayy=2aac\end{matrix}\}$

Erit ex $\begin{matrix}1^{mâ}\\2^{dâ}\end{matrix}\begin{cases}x=\frac{aby+aab}{bb-ay}\\x=\frac{2aac-ayy}{bc}\end{cases}$

Quibus valoribus quantitatis x, inter ſe æquatis, & terminis rite ordinatis ac diſpoſitis, emergit æquatio pro invenienda quantitate y, viz.

$$y^3-\frac{bb}{a}yy-\frac{2aac-bbc}{a}xy+bbc=0.$$

Eodem modo $\begin{matrix}\text{ſi}\quad evv+eev=x^2\\ \&\quad vx-ev=ex\end{matrix}\}$

Erit

Erit $v = \frac{ex}{x-e}$, ex 2da Æqu. & ſubſtituendo expreſſionem hanc loco ipſius v in 1ma, erit $\frac{e^2xx}{x^2-2xe+ee} + \frac{e^2x}{x-e} = x^2$; unde multiplicando per $x^2-2xe+ee$, habemus $2e^2xx-e^4x = x^4-2x^3e+x^2ee$ vel ordinando & reducendo, $x^4-2x^3e+xxee-2xe^3+e^4 = 0$.

Sit etiam $zvv = c^3$
& $zz+vv+az = dv$ }

erit $vv = \frac{c^3}{z}$ adeoq; $zz+\frac{c^3}{z}+az = d\sqrt{\frac{c^3}{z}}$, hoc eſt $z^3+c^3+azz = d\sqrt{c^3z}$, unde orietur æquatio ſex dimenſionum, qua exprimetur terminus z. Sed majori compendio hoc fiet, ſi valorem, non ipſius vv, ſed z, deſumptum ex prima, ſcribamus in ſecunda æquatione. Nam cum $z = \frac{c^3}{vv}$, ergo $\frac{c^6}{v^4}+vv+\frac{ac^3}{vv} = dv$, & facta Reductione, $c^6+v^6+ac^3vv = dv^4$, ſive $v^6-dv^4+ac^3vv+c^6 = 0$.

Adeo ut vel ex hoc ſolo exemplo liquet, quantum valet ac confert *ad expreſſionis ſimplicitatem, apta & commoda ſubſtitutio.*

Et ex hiſce Exemplis, ad alia progredi liceat, ubi ſine æquationibus multo magis complicatis, iiſque interdum ad infinitum uſque excurrentibus, res conficienda non erit.

Ut

Ut ſi $\left.\begin{array}{l} \frac{x^n}{m} - \frac{y}{n} = a \\ myq + z^p = b \\ \frac{z^m}{n} - v^r = c \\ x^r - \frac{v^m}{p} = d \end{array}\right\}$

Nam extrahendo utrinque Radic. n & r,

Erit ex 1ma 4ta $\left\{\begin{array}{l} \sqrt[n]{ma + \frac{my^m}{n}} = x \\ \sqrt[r]{d + \frac{1}{p}v^m} = x \end{array}\right.$

Et elevando hæc æqualia ad Poteſt. r (ex. gr.) erit

$\sqrt[n]{ma + \frac{my^m}{n}}^r - d = \frac{1}{p}v^m$, vel $v =$

$\sqrt[m]{\frac{\sqrt[n]{ma + \frac{my^m}{n}}^r - d}{\frac{1}{p}}}$; quæ *prima* eſt expreſſio incog-

nitæ v, in terminis y, & quantitatum *datarum*.

Rurſus ex 2da 3^{tia} $\left\{\begin{array}{l} \sqrt[p]{b - my_q} = z \\ \sqrt[m]{nc + nv^r} = z \end{array}\right.$

&

& operando ut prius, invenies *secundam* expressionem ipsius v in y & quantitatibus cognitis,

$$viz.\ v = \overline{\frac{\overline{b - my^p}|^{\frac{m}{p}} - nc}{n}}|^{\frac{1}{r}}.$$

Unde, series (*Finita* vel) *Infinita* (exponentes enim diversis modis sumi possunt) *constans ex potestatibus* incognitæ y, & quantitatibus datis, æqualis erit alteri seriei, (*Finitæ* vel) *Infinitæ, ex potestatibus ipsius y & quantitatibus datis*, similiter constanti. Et per consequens orietur æquatio, quâ quantitas incognita y, plenè determinatur ac exprimitur. Et hinc difficile non erit æquationes pro cæterarum incognitarum valoribus determinandis, invenire.

Imo hoc ipsum fieri potest quando quantitates incognitæ ita disponuntur, *ut quædam ex ipsis, per series infinitas in aliarum terminis prius explicari debeant*, quàm æquatio pro unâ incognitâ, se prodat. Supponantur enim æquationes,

$$\left.\begin{array}{l} 1+z = \overline{1+y}^{n} \\ 1+y = \overline{1+x}^{m} \\ 1+x = \overline{1+v}^{r} \\ 1+v = \overline{1+z}^{s} \end{array}\right\}$$

Tum comparando duos valores ipsius z. Ex equationibus 1ma & 4ta acceptos.

$$\text{Ex } \begin{matrix} 1^{ma} \\ 4^{ta} \end{matrix} \left\{ \text{erit } \begin{matrix} z = \overline{1+y}^{n} - 1 \\ z = \overline{1+v}^{s} - 1 \end{matrix} \right\} \begin{matrix} \text{adeoque} \\ \overline{1+y}^{n} = \overline{1+v}^{s}. \end{matrix}$$

Vel

Vel poteſtates haſce per proprias ſuas ſeries exprimendo, erit $1+ny+n\times\frac{\overline{n-1}}{2}y^2+n\times\frac{\overline{n-1}}{2}\times\frac{\overline{n-2}}{3}y^3$, &c. $=1+\frac{1}{s}v+\frac{1}{s}\times\frac{\overline{1-s}}{2s}vv+\frac{1}{s}\times\frac{\overline{1-s}}{2s}\times\frac{\overline{1-2s}}{3s}v^3$, &c. Et per conſequens *habebitur ſeries ex poteſtatibus y compoſita, æqualis ſeriei compoſitæ ex poteſtatibus ipſius v.* Sed per Methodum pro Extractione Radicis ex Æquatione Infinitâ (quam Publico communicavit *vir Clariſſimus* D. De *Moivre*) quantitas y exprimi poteſt per Poteſtates ipſius v.

Nempe erit $y=\frac{1}{ns}v+\frac{1}{s}\times\frac{\frac{\overline{1-s}}{2s}-\frac{\overline{n\times\frac{\overline{n-1}}{2}}AA}{n}vv+\frac{1}{s}\times\frac{\overline{1-s}}{2s}\times\frac{\overline{1-2s}}{3s}-\frac{\overline{n\times\overline{n-1}}AB-\frac{\overline{n\times\frac{\overline{n-1}}{2}\times\frac{\overline{n-2}}{3}}A^3}{n}v^3}{n}$

&c. Ubi litera $A=\frac{1}{ns}$, $B=\frac{1}{s}\times\frac{\frac{\overline{1-s}}{2s}-\frac{\overline{n\times\frac{\overline{n-1}}{2}}AA}{n}}{n}$,

& ſic deinceps; ponendo ſemper pro literis Capitalibus, B, C, D, E, &c. coefficientes Totas terminorum immediatè precedentium. Adeoque ſic aliter explicari poteſt valor ipſius y, nempe $y=\frac{1}{ns}v+\frac{\overline{1-ns}}{2s^2n^2}vv+$ &c.

Eodem

Eod. Modo, ex æq. $\left.\begin{array}{ll} 2^{da} & x = \overline{1+v}^{r} - 1 \\ 3^{tia} & x = \overline{1+y^{\frac{1}{m}}} - 1 \end{array}\right\}$ adeoque $\overline{1+v}^{r} = \overline{1+y^{\frac{1}{m}}}$.

Unde erit $1+rv+r\times\frac{r-1}{2}v^2+r\times\frac{r-1}{2}\times\frac{r-2}{3}v^3$

&c. $= 1+\frac{1}{m}y+\frac{1}{m}\times\frac{1-m}{2m}yy+\frac{1}{m}\times\frac{1-m}{2m}\times\frac{1-2m}{3m}y^3$

&c.

Quare series ex potestatibus ipsius y composita, æqualis erit serei, ex potestatibus incognitæ v, similiter constanti. Et ope *Methodi prædictæ*, exprimetur y in terminis ipsius v. Hoc est,

$$y = mr\times v+r\times\frac{r-1}{2}\times\frac{\frac{1}{m}\times 1-m}{2m}AA\,vv+$$

$$r\times\frac{r-1}{2}\times\frac{r-2}{3}\times\frac{\frac{1}{m}\times 1-m}{m}AB-\frac{1}{m}\times\frac{1-m}{2m}\times\frac{1-2m}{3m}A^3\,v^3$$

&c. vel etiam $y = mr\times v+\frac{m^2r^2-mr}{2}vv$, &c.

Quapropter cum habeatur y bis expressa in terminis v; ex duabus hisce expressionibus conflabitur æquatio desiderata quâ ipsa quantitas v determinatur:

minatur; unde patet via ad ipſas z, x, & y inveniendas.

Et via conſimili, in plurimis aliis Caſibus, ubi modis longe diverſis, quantitates incognitæ aliæ aliis implicantur, earum expreſſiones obtineri poterunt. Sed hæc nec ad Praxin vulgarem pertinent; neque Tironum captui accommoda ſunt, ac proinde non ſunt hujus loci, verum haud incongruum fore duxi, ſi data occaſione, pauca quædam, de re non prorſus inutili, per tranſennam quaſi, dicerem.

Et hactenus de modo inveniendi æquationes, quarum ope quantitates incognitæ exprimantur, *cum ipſæ Pares numero cum æquationibus exiſtunt.*

Si vero plures ſint incognitæ, quam æquationes, tunc aliquæ earum (uti prius diximus) ultimo manebunt. Nempe ſi ſint *una* plures, *duæ* manebunt, ſi *duabus*, *tres*, ſi *tribus quatuor*, & ſic deinceps. Quod tum ex Natura rei, tum per exempla manifeſtum fiet. Sint enim æquationes, $\frac{xy}{c} = z$, & $cy + zx = cc$. Tunc ſubſtituendo pro x, vel y, vel z, in ſecunda æquatione, vel $\frac{cz}{y}$, vel $\frac{cz}{x}$, vel $\frac{xy}{c}$, ex Primâ; orietur vel $cy + \frac{czz}{y} = cc$, vel $\frac{ccz}{x} + zx = cc$, vel $cy + \frac{xxy}{c} = cc$; in quibus ſingulis, incognitarum ultimò manentium numerus eſt 2, *viz. Aggregatum unitatis*, & exceſſus Numeri incognitarum quantitatum, ſupra Numerum æquationum, prior enim fuit 3 & poſterior 2.

Similiter ſi $\frac{yy}{z} - 3a = 3x$, $av + vx = 5zz$, $cc - yy = bv$. Erit primò $yy = 3zx + 3az$, Secundo $yy = cc - bv$, $\therefore 3zx + 3az = cc - bv$, & $\frac{cc - bv}{3z} = a + x$; & ſubſtituendo $\frac{cc - bv}{3z}$ loco $a + x$ (in æquatione $av + xv = 5zz$) erit $ccv - bvv = 15z^3$; ubi duæ tantum manent incognitæ.

At ſi harum æquationum duæ ſolummodo tentandæ eſſent: *ex. gr. Prima* & *Ultima*, tunc in æquatione $3zx + 3az - cc + bv = o$, tres ultimò manerent incognitæ, *viz.* x, z, & v. Vel ſumptis æquationibus *ſecunda*, & *ultima*, eſſet $\frac{5zz}{a + x} (= v) = \frac{cc - yy}{b}$, vel $5bzz - acc + ayy - xcc + xyy = o$; in quâ tres etiam ſunt incognitæ.

Ultimò, *ſi plures ſint æquationes quam incognitæ*, orientur Caſus (ſicut prius innuimus) vel *impoſſibiles*, vel ad *certas quaſdam proportiones*, quantitatum Equationes componentium, *determinati ac reſtricti*. Quod Exemplo uno vel altero dilucidum fiet.

Ut ſi ponatur
$$\left.\begin{array}{l} a + 3z = b \\ x - 2b = 4a \\ x + z = 2a - b \end{array}\right\}$$

Erit

Erit (ex 3^tia) $x = 2a - b - z$, & (ex 2da) $x = 4a + 2b$; $\therefore 4a + 2b = 2a - b - z$; sed (ex 1ma) $z = \frac{b-a}{3}$, $\therefore 4a + 2b = 2a - b - \frac{-b+a}{3} = \frac{7a-4b}{3}$, adeo ut jam sit $\frac{7a-4b}{3} = 4a + 2b$, vel $7a - 4b = 12a + 6b$, & $7a = 12a + 10b$. Quod absurdum est & impossibile.

Supponatur rursus $\left.\begin{matrix} a - y = d \\ 3y - a = c \end{matrix}\right\}$

erit $a - d = y = \frac{c+a}{3}$, unde $3a - 3d = c + a$, vel $2a - 3d = c$. Ac propterea, si posità *prima* æquatione, quantitas c in *secunda*, ad hunc valorem restricta fuerit) congruæ omnino, ac sibi mutuo consistentes, erunt illæ duæ ab initio factæ Hypotheses. Nam quoniam ex *prima* æquatione sit $a - y = d$, erit etiam $3a - 3y = 3d$. Sed ex *secunda* est $3y - a = c$, & cum c (ex Hypothesi) $= 2a - 3d$, erit etiam $3y - a = 2a - 3d$, vel $3y = 3a - 3d$ vel mutando signa, $3a - 3y = 3d$; quæ æquatio prorsus eadem est cum priore.

Et ex hisce forsan Tyrones lucem aliquem accipere possint, cujus ope in horum generum æquationibus tractandis, non sine commodo dirigantur. Adhuc vero opus iis quam gratissimum, me præstiturum fore confido, si Regulas illas elegantes pro exterminandis quantitatibus incognitis, quas dedit Celeberrimus D. *Newtonus*, huc transferam, & propriis ipsius verbis expressas (quasi in Synopsi) iis exhibeam.

Reg. I. *Cum Quantitas tollenda unius eſt tantum dimenſionis in utraque æquatione, valor ejus uterque, per Regulas* (Reductionis nempe) *ante traditas, quærendus eſt, & alter valor ſtatuendus æqualis alteri.*

Reg. II. *Cum in altera ſaltem æquatione, tollenda quantitas unius tantum dimenſionis exiſtit, valor ejus in ea quærendus eſt, & pro ſe in æquationem alteram ſubſtituendus.*

Reg. III. *Cum in neutra æquatione tollenda quantitas unius tantum dimenſionis exiſtit, valor maximæ poteſtatis ejus in utraque quærendus eſt; deinde ſi poteſtates iſtæ non ſint eædem, æquatio poteſtatis minoris multiplicanda eſt per tollendam quantitatem aut per ejus Quadratum aut Cubum, &c. ut ea evadat ejuſdem poteſtatis cum æquatione altera. Tum valores illarum poteſtatum ponendæ ſunt æquales, & æquatio nova prodibit ubi maxima poteſtas ſive dimenſio tollendæ quantitatis diminuitur. Et hanc operationem iterando, quantitas illa tandem auferetur.*

Quæ exemplis præcedentibus facillime applicari, & in Praxin deduci poſſunt.

CAP.

CAP. IV.

De Natura Æquationum.

SUB Æquationum involucris, latent Problematum Solutiones, quæ per *Analyſin* vel Radicum extractionem denudantur & ob oculos ponuntur. Ut vero modus tractandi æquationes, earum *ſolutioni* & *conſtructioni* inſerviens, melius intelligatur, totaque poſtea tradenda Praxis ſtabiliori inſiſtat Fundamento; rem ab ipſo initio repetemus, de Geneſi Æquationum primo loco differentes.

Generantur (vel ſaltem generari concipiendæ ſunt) æquationes, per multiplicationem quorundam Binomiorum, in quibus quantitas aliqua *quæ ut incognita ſpectatur*, cum aliis quæ cognitæ ſunt per ſigna + & — pro lubitu conjungitur.

Ut ſi fingamus incognitam z ambiguè ſignificare quemlibet ex his duobus numeris, *viz.* 4 & 5, adeo ut ſit $z=4$, $z=5$, vel $z-4=0$, & $z-5=0$. Hiſce ergo in ſe multiplicatis, ſit æquatio $zz-5z-4z+20=0$, vel $zz-9z+20=0$, colligendo terminos in quibus z reperitur, in unam ſummam.

Similiter ſi exponamus ipſam z per c & d, vel per $-c$ & $-d$, vel c & $-d$, ſic ut fiat

$$\left.\begin{matrix} z-c=0 \\ z-d=0 \end{matrix}\right\} \text{ vel } \left.\begin{matrix} z+c=0 \\ z+d=0 \end{matrix}\right\} \text{ vel denique}$$

$\left.\begin{matrix} z-c=0 \\ z+d=0 \end{matrix}\right\}$ erit in *Prima* Hypotheſi, $zz-zc+zd-dc=0$, in ſecunda $zz+zc+zd+dc=0$, in Tertiâ $zz-zc+zd-dc=0$. Hoc eſt ponendo $d+c=q$, $d-c=+p$, prout d eſt, $>$ vel $>c$; orietur $zz-qz+dc=0$, vel $zz+qz+dc=0$, vel $zz+qz-dc=0$.

Eodem modo, ſi $x=a$, $x=b$, $x=-c$, $x=d$, &c. ſive $x-a=0$, $x-b=0$, $x+c=0$, $x-d=0$, &c. Tunc facta multiplicatione $x-a$ per $x-b$, emergit æquatio $xx\begin{matrix}-a\\-b\end{matrix}x+ab=0$. Et hac rurſus multiplicata per alterum Binomium $x+c$, orietur æquatio uno gradu altior, nempe

$$x^3\begin{matrix}-a\\-b\\+c\end{matrix}xx\begin{matrix}+ab\\-ac\\-bc\end{matrix}x+abc=0.$$

Ulterius multiplicando hanc ultimam per $x-d$, producetur æquatio uno gradu adhuc ſublimior quæ eſt

$$x^4\begin{matrix}-a\\-b\\+c\\-d\end{matrix}x^3\begin{matrix}+ab\\+ac\\-bc\\+ad\\+bd\\-cd\end{matrix}xx\begin{matrix}-abc\\+abd\\+bcd\\+acd\end{matrix}x-abcd=0$$

Et Novos adhuc fingendo valores Quantitatis Incognitæ x, eoſque vel *Poſitivos* vel *Negativos*, prout libuerit; in æquationibus magis compoſitis Contexendis, uſque ad Infinitum procedere potes.

Ex

Ex hac vero earum Geneſi, ſequentia facile colliguntur.

1*mo. Singulis Multiplicationibus, uno gradu augetur altiſſima Poteſtas quantitatis Incognitæ.*

Ex *prima* ergo Multiplicatione quantitas incognita aſcendit ad *Quadratum*, unde ipſa æquatio *Quadratica* appellatur; ex *ſecunda* ad *Cubum*, ex *tertia* ad, *Quadrato Quadratum*, &c. unde æquatio ſimiliter *Cubica* aut *Bi-quadratica*, &c. nominatur.

2*do. Tot poſſunt eſſe diverſi valores quantitatis incognitæ* (vel æquationis Radices) *quot ipſa incognita habet dimenſiones, & non plures.*

Patet hoc ex Generatione.

3*tio. Omnium ſimul Terminorum aggregatum, nihilo æquale eſt.*

Ex Hypotheſi enim ſingula *Binomia Generatrix* hoc eſt ſinguli Factores aggregati producti, nihilo æquales ſunt; adeoque in ſe invicem ducti nihilum efficiunt.

4*tio*, Patet ſi *qualibet* ex *primo aſſumptis quantitatibus, ubique in locum incognitæ ſubſtituatur, quod Aggregatum omnium Terminorum manebit adhuc nihilo æquale.*

Ut in prædictâ æquatione Cubicâ,

$$x^3 \begin{matrix} -a \\ -b \\ +c \end{matrix} xx \begin{matrix} +ab \\ -ac \\ -bc \end{matrix} x + abc = 0;$$

Ubi fuit $x=a$, vel b, vel $-c$. Si ejus loco ponatur a.

$$\text{orietur } a^3 \begin{matrix} -a & +ab \\ -baa & -ac \\ +c & -bc \end{matrix} \quad a+abc=0, \text{ ſi } b,$$

$$\text{erit } b^3 \begin{matrix} -a & +ab \\ -b\ bb & -ac\ b \\ +c & -bc \end{matrix} +abc=0, \text{ ſi vero } -c,$$

$$\text{erim tum } -c^3 \left.\begin{matrix} -a & +ab \\ -b\ cc & -ac \\ +c & -bc \end{matrix}\right\} \times c+abc=0.$$

Quod manifeſtum eſt, tum *ex ipſa terminorum Diſpoſitione (quæ talis eſt ut ſe mutuo deſtruant omnes)* tum etiam ex hâc conſideratione, *quod cum tota ſumma prius fuerit ex Generatione nihilo æqualis, impoſſibile ſit æqualem ſubſtitutionem* ullam mutationem efficere. Hoc idem vero adhunc modum aliter probari poterit.

Quoniam enim (ex *Geneſi*) habetur

$x^3+c-a-b\times xx+ab-ac-bc\times x+abc=0$, ergo dividendo Totum per x^3, erit

$$1+c-a-b\times\frac{1}{x}+ab-ac-bc\times\frac{1}{xx}+\frac{abc}{x^3}=0;$$

& Multiplicando per eandem Poteſtatem (in hoc exemplo *Cubum*) cujuſvis ex iis cui x æqualis ponebatur, ex. gr. per $-c3$, producetur

$$-c^3+c-a-b\times\frac{-c^3}{x}+ab-ac-bc\times\frac{-c^3}{xx}+$$

abc

$\frac{abc}{x^3} \times -c^3 = 0$. Sed idem per hanc Multiplicationem efficitur, ac ſi loco ipſius x poneretur $-c$ per Totam æquationem. Nam quoniam $x = -c$, eſt $\frac{-c^3}{x} = +c^2$, & $\frac{-c^3}{xx} = -c$, & $\frac{-c^3}{x^3} = 1$, unde his in illorum locos ſubſtitutis, erit ſumma omnium, $-c^3 + c - a - b \times c^2 + ab - ac - bc \times -c + abc$, quæ nihilo æqualis eſt, hoc eſt

$$\begin{array}{llll} -c^3 & -a & +ab & \\ & -b\,cc & -bc \times -c & +abc = 0, \text{ ut prius.} \\ & +c & -ac & \end{array}$$

5to. Radix ergo æquationis alicujus eſt, quæ (ſive Poſitiva ſive negativa ſit) in locum quantitatis incognitæ ubique ſubſtituta, facit ut Tota ſumma Evaneſcat, vel nihilo æqualis evadat.

Quantitates enim quæ hoc efficiunt, eæ ipſæ ſunt quas incognita x primò ambiguè ſignificabat; hoc eſt, ſunt ejus *diverſi valores*, vel (alio vocabulo) generatæ æquationis *radices*. Præter has vero, nullæ aliæ quantitates hoc idem præſtare poſſunt. Ex. gr. in æquatione Numericâ $xx - 7x + 12 = 0$, ex factoribus $x - 3$, & $x - 4$, productâ, ubi per conſequens incognita x per numerum 3 vel 4 explicanda eſt; ſi ponas 4 loco x, erit tum $16 - 28 + 12 = 0$, vel ſi 3, erit $9 - 21 + 12 = 0$: Ponatur vero 5, & tunc erit $25 - 35 + 12 > 0$, ſi 2, erit $4 - 24 + 12 < 0$, ſi 1, erit $1 - 7 + 12 > 0$. Sicque alia fingi exempla, ad aliiique libitum numeri tentari poſſunt.

6to.

6to. Mutatis ſignis in Locis Paribus, reliquis immutatis, coefficientes ſive Quantitates cognitæ Termini Secundi, Tertii, Quarti, Quinti, &c. exhibent reſpective ſummas ipſarum Radicum, Rectangulorum ſub binis, ſolidorum ſub ternis, contentorum ſub quaternis, &c. Notandum vero, quod ultimus Terminus ſemper ſit productum (ſive Rectangulum, ſive ſolidum, &c.) ſub omnibus radicibus. Ponatur enim $y=p$, $y=q$, $y=-r$, $y=-s$, Et ex hiſce Factoribus $y-p=0$, $y-q=0$, $y+r=0$, $y+s=0$, generetur æquatio.

$$\begin{array}{llll} -p & +pq & & \\ -q & -pr & +pqr & \\ y^4 +r\ y^3 & -qr\ y^2 & +pqs\ y + pqrs = 0. & \\ +s & -ps & -qrs & \\ & -qs & -prs & \\ & +rs & & \end{array}$$

Termini 2di. Coefficiens, prout ſignis hic afficitur, eſt $r+s-p-q$, & ſignis, in contraria mutatis, ſit $p+q-r-s$, quæ eſt ipſa Radicum ſumma ſubſignis propriis. Nam r & s fuerunt quantitates negativæ ex primâ ſuppoſitione. At vero $pq+rs-pr-qr-ps-qs$ Quantitas cognita 3tii. Termini, eſt ſumma rectangulorum ſub binis quibuſcunque radicibus cum ſuis propriis ſignis; duo enim ex his, *viz.* pq & rs, *ab origine affirmativi* ſunt quoniam producuntur ex multiplicatione affirmativi in affirmativum & negativi in negativum; & alii quatuor *negativi* ſunt, propter multiplicationem affirmativi in negativum, ex quâ oriuntur. Similiter mutatis ſignis de $pqr+pqs-qrs-prs$, ſit $qrs+prs-pqr-pqs$, aggregatum ſolidorum

lidorum ſub ternis quibuſcunque radicibus iiſque ſuis propriis ſignis affectis: Sunt enim ex his ſolidis etiam duo quæ *ab origine ſigna habent affirmativa*, *viz.* *qrs* & *prs*, quoniam oriuntur ex multiplicatione affirmativi in affirmativum (*viz.* *p* & *q* in *rs*) alia duo *pqr*, *pqs* *negativa* ſunt, quoniam è contrario producuntur ex multiplicatione negativi in affirmativum, *viz.* *pq* in *r* & *s*. *Ultimus* vero terminus *pqrs*, (*viz.* contentum ſub omnibus radicibus) ſignum habet poſitivum, quo etiam *ab origine* afficitur, propter multiplicationem affirmativi *pq* in affirmativum *rs*.

Et ut verbo dicam; attende ad ſigna, quibus quantitates ab origine affectas ſupponis, & ad eorum mutationes in contraria, dum Binomia componis ut inſtituatur multiplicatio; & hiſce bene perpenſis, horum omnium rationes immò & demonſtrationes te latere non poſſunt.

7mo. Si membra affirmativa & negativa, Quantitatem cognitam alicujus Termini æquationis conſtituentia, inter ſe fuerint æqualia, ſe mutuo deſtruent, & per conſequens Terminus ille penitus evaneſcet.

Et vice verſa ſi Terminus aliquis æquationis abfuerit, neceſſe eſt ea membra eſſe inter ſe æqualia.

Adeoque Termino *ſecundo*, ex. gr. tunc carebit æquatio, cum radices poſitivæ ſimul ſumptæ æquales ſunt ipſis negativis etiam ſimul ſumptis. Et ſic de cæteris.

8vo. Æquationis ſumma ſemper dividi poterit per Binomium aliquod, compoſitum ex Quantitate incognita minus valore alicujus ex radicibus affirmativis, vel plus valore alicujus ex negativis.

Nec

Nec per ullum aliud Binomium, unquam dividi poterit æquatio propoſita.

Præterea dictæ diviſionis ope, æquationis dimenſiones in tantum minuentur. Ex. gr. Dividendo *Ultimam Quatuor dimenſionum* æquationem per $y-p$, ea evadet *Cubica*, unam *affirmativum*, & duas negativas radices complexa; & rurſus factâ diviſione hujus *Cubicæ* per $y+r$, orietur *Quadratica*, in quâ una ſolum negativa & una affirmativa continentur: & hâc ipſâ Quadratica iterum divisâ per $y+s$, reſultat æquatio ſimplex, $y-q=0$. Eodem etiam modo, æquatio $x^3-6x^2-51x+280=0$, ex factoribus $x-5=0$, $x-8=0$, $x+y=0$ generata, per horum quamlibet, ſine reſiduo dividi poteſt, at non per $x+$ vel $-$ quolibet alio numero. Ex. gr. dividendo eam per $x+7$ orietur $xx-13x+40=0$, æquatio continens duas reliquas radices, affirmativas nempe 8 & 5; quâ rurſus divisâ per $x-5=0$, vel $x-8=0$, habebitur in Quotiente $x-8=0$, vel $x-5=0$.

Hinc, ſi æquationis ſumma dividi non poterit per Binomium compoſitum ex quantitate incognitatâ $+$ vel $-$ aliâ quâdam quantitate datâ; quantitas illa data valor non eſt ullius ex radicibus iſtius æquationis.

9no. *Patet etiam ex Geneſi, quomodo in æquationibus* (quarum omnes radices *Reales* ſunt) *numerus radicum tum Affirmativarum tum Negativarum, definiri queat.*

Tot enim ſunt *Affirmativæ*, quot ſignorum in ſerie continuâ mutationes de $+$ in $-$, & $-$ in $+$, & per conſequens cæteræ ſunt *Negativæ*: vel (quod idem eſt) tot ſunt negativæ, quot ſunt ſignorum paria $-$ & $-$ vel $+$ & $+$, ſe invicem imme-

immediatè ſequentia. Sic in æquatione $x^3 - 6x^2 - 51x + 280 = 0$, duæ ſunt affirmativæ radices, propter duas ſignorum, mutationes, *viz. de* + in −, in terminis x^3 & $6x^2$, & *de* − in +, in terminis $51x$ & 280: Una etiam eſt negativa, ut ſine reſpectu ad ſigna ſatis manifeſtum eſt.

Et per eandem regulam, in æquatione $z^4 - 4z^3 - 19z^2 + 106z - 120 = 0$, tres ſunt affirmativæ, et una tantum negativa: *tres enim ſunt ſignorum variationes, & una tantum eorundem ſignorum immediata ſucceſſio.* Et horum omnium rationes, ex Algebraicæ multiplicationis natura, nullo negotio deducent ipſi Tyrones.

10*mo. Denique ex eodem fonte, manat regula pro Tranſmutandis æquationis radicibus, Affirmativis in Negativas, & viciſſim Negativis in Affirmativas.*

Nempe mutando ſigna in locis alternis (hoc eſt 2*do*, 4*to*, 6*to*, *&c.* vel 1*mo*, 3*tio*, 5*to*, *&c.*) reliquis prorſus immutatis.

Ex gr. æquatio $z^4 - 4z^3 - 19z^2 + 106z - 120 = 0$, continet tres radices affirmativas, 2, 3, 4, & aliam negativam, −5. Mutatis vero ſignis in locis paribus, evadit $z^4 + 4z^3 - 19z^2 - 106z - 120 = 0$, in qua ergo vi regulæ prioris, tres ſunt negativæ, *viz.* −2, −3, −4, (eædem quæ prius erunt affirmativæ) & una Affirmativa quæ 5 eſt (quæ etiam in priore Negativa fuit) Hoc eſt, per hanc operationem, in eam ipſam formam mutatur æquatio, ſub qua compareret, ſi ſumptis factoribus $z + 2 = 0$, $z + 3 = 0$, $z + 4 = 0$, $z - 5 = 0$, via hactenus oſtenſa generanda eſſet. Idem vero eveniet ſimutentur ſigna in locis Imparibus, adeoque pro $z^4 - 4z^3 - 19z^2 + 106z - 120 = 0$, ſcribatur

$-z^4$

$-z^4-4z^3+19z^2+106z+120$. Hic enim eædem tres ſunt Radices negativæ, & eadem affirmativa quæ priùs; ſolummodo tranſponuntur membra factores componentia, ut loco $z+2$, $z+3$, $z+4$, $z-5$, fiant $2+z$, $3+z$, $4+z$, $5-z$, unde altiſſima Poteſtas ſigno fruetur negativo.

Schol. Ad ulteriorem explicationem *naturæ Radicum*, obſervetur diſtinctio earum maximè Generalis qua in *Reales* (ſive Poſſibiles) & *Imaginarias* (ſive *Impoſſibiles*) dividuntur. *Realium* rurſus aliæ ſunt *Affirmativæ*, aliæ vero *Negativæ*, de quibus (& de quibus ſolis) hactenus locut fuimus. Quod ad *Impoſſibiles* ſpectat, ſciendum eſt, eas nec Affirmativas nec Negativas eſſe, ac proinde nihil eſt omnino, quod intellectus de iis concipere valeat. Omnis vero Impoſſibilitas, ex *Quadrato negativo*, ortum ſuum trahit, & in hoc fundatur.

Nulla enim datur Quantitas, quæ ullo modo (a nobis) vel cogitari vel explicari poterit, quin ſi in ſeipſam ducatur, ſive ea ſit Negativa ſive Affirmativa, faciet *Quadratum* Affirmativum. Radix ergo Quadrati Negativi, Fictitium Quid eſt & Impoſſibile: ſicut etiam ſunt Radices omnium Poteſtatum Negativarum quæ per numeros Pares deſignantur, utpote ad *Quadrata reducibilium.*

Jam hinc oritur limitatio neceſſaria, ad unam ſaltem è regulis prius allatis ſpectans, ac ſine quâ ut univerſaliter vera nullo modo recipienda eſt. Nimirum, *quando æquationis nullæ radices ſunt impoſſibiles, tunc de numero radicum affirmativarum ac negativarum per æquationis ſigna tuto judicari poterit.* Cum aliter fieri poſſit, ut tot affirmativæ non habeantur, quot ſunt variationes ſignorum + & −;

nec

nec tot negativæ quot vicibus duo ſigna + vel duo — ſe invicem ſequentia deprehenduntur. Radices enim Impoſſibiles nec ſunt affirmativæ nec negativæ, uti prius oſtenſum eſt: adeoque cum æquatio earum aliquas incluſerit, *regula valere non poteſt*, niſi quatenus eas pro *ambiguis* æſtimaveris. Ut hæc melius intelligantur ſciendum eſt, æquationes generari poſſe per multiplicationem continuam, non ſolum quantitatum *Realium* & Poſſibilium, ſed etiam Impoſſibilium & *fictitiarum*. Sicut ſi ponas $x=2+\sqrt{-1}$, & $x=2-\sqrt{-1}$ (ubi *quantitas impoſſibilis eſt radix unitatis negativa*) Tum multip. $x-2-\sqrt{-1}=0$ per $x-2+\sqrt{-1}=0$ orietur productum $x^2-2x-x\sqrt{-1}-2x+4+2\sqrt{-1}+x\sqrt{-1}-1-2\sqrt{-1}+1=0$, vel rejectis rejiciendis, $x^2-4x+5=0$, quæ ergo eſt *æquatio impoſſibilis*. Hâc vero rurſus ductâ in $x-2=0$, *ex. gr.* producetur æquatio $x^3-6x^2+13x-10=0$; cujus ſigna ſi ſpectes, prima fronte apparebunt tres radices affirmativæ: at vero earum una tantum eſt *realis*, & duæ reliquæ (ut jam vidiſti) *imaginariæ ſunt & impoſſibiles*.

Sic æquatio $x^3-4xx+7x-6=0$, (cui etiam ex ſignorum variationibus tres radices affirmativas indubius protinus aſcriberes) oritur ex radicibus 2, $1+\sqrt{-2}$, $1-\sqrt{-2}$, quarum duæ poſteriores eâdem Impoſſibilitatis notâ inſigniuntur.

Et hinc etiam ulterius concludi poterit, quod non univerſaliter verum ſit, *tot eſſe radices æquationis alicujus, quot ſunt dimenſiones altiſſimæ poteſtatis*; niſi radicis notionem extendas *ad omne id* (ſive poſſibile ſive impoſſibile) *quod ad formationem*

nem æquationis aliquo modo concurrere, mente tua concipis. Et ſane verum eſt ipſas radices impoſſibiles (non minus quam cæteræ) in locum incognitæ ubique ſubſtitutas totam ſummam nihilo æqualem relinquere, ut periculum facienti manifeſtum erit. Si vero id tantùm per radicem velis, *quod explicari vel intelligi poſſit*, tunc limitationem regula ſubire debet, cum aliter liquidò appareat eam in caſibus innumeris falſam ac fallacem eſſe.

Unum vero vel alterum adhuc monendum reſtat de radicibus hiſce impoſſibilibus; in quibus *magnum illum Newtonum* (in Tractatu prius laudato) ſequimur.

1mo *Cum Caſus Problematum, interdum ſint poſſibiles, in aliis vero circumſtantiis penitus impoſſibiles; propter hanc rationem, æquationum radices aliquando debent eſſe impoſſibiles; aliter enim omnia Problemata tanquam poſſibilia, in omni Caſu, ſine diſcrimine repreſentarent.*

Sic conſtruendo æquationem $zz - az + bb = 0$, ubi circulus deſcribitur radio $= \frac{1}{2}a$, & in Tangente ponitur longitudo ipſius b, ad cujus extremitatem linea perpendicularis erigenda eſt. Hæc producta, circulum in duobus punctis *ſecabit*, quamdiu *minor radio* manet Tangentis longitudo: *Tanget vero cum æquales ſunt*, & *nunquam occurret, cum poſterior priorem excedit.* Hoc plane modo, per æquationem, quâ valor incognitæ z exprimitur (nempe $z = \frac{1}{2}a + \sqrt{\frac{1}{4}aa - bb}$) *conſtructionis poſſibilitas vel impoſſibilitas* vere ac juſte exhibetur. Prout enim ponitur $\frac{1}{2}a >$ vel $< b$, ita fit $\frac{1}{4}aa - bb$ Quadratum vel *Affirmativum*

mativum vel *Negativum*, ac per conſequens ejus Radix (& ſic tota z) vel poſſibilis vel impoſſibilis evadit.

2do *Æquationum radices ſæpenumero in ſe reales ſunt ac Poſſibiles, cum omnem lata* Figuris Geometrica *explicationem reſpuunt*; hoc eſt cum per Schemata ſunt impoſſibiles.

Cujus rei ratio eſt, quod relationes eæ, quibus concipere poſſumus quantitates in æquatione, ſe mutuò reſpicere; nimis abſtractæ ſunt acgenerales, quam ut ſub anguſtis Figurarum limitibus coerceantur.

Ut ſi proponatur hæc æquatio $\frac{y^*}{a} = x$. In terminis hiſce abſtractis, nil aliud ſibi vult hæc æquatio, niſi quod x ſit tertia proportionalis duabus y & a. Et hoc ſemper poſſibile erit, quantitate a *invariata* manente, & *variante* immo creſcente y in Infinitum. Si vero hanc ipſam æquationem explicare velis, per relationem quarundam linearum in figura Geometrica, ex. gr. *Circulo*, ponendo *Diametrum*, a, *Chordam* ex ejus termino ductam, y; erit x, *Abſciſſa* chordæ reſpondens.

Et hic tibi ponuntur limites a naturâ circuli in determinandâ magnitudine quantitatis y, ultra quos excurrere non licet: neceſſariò enim minor erit quam a, aliter nullam omnino 3*tiam* proportionalem, vel abſciſſam x, per circulum unquam obtinebis.

3^tio Sed notandum, *quod ſicut hæc Impoſſibilitas ortum ſuum trahit, ex natura & proprietate certæ cujuſdam figuræ; ſic adhibendo figuram diverſæ naturæ, fieri poſſit ut nulla omnino Impoſſibilitas reperiatur, ſed* deſignatio Geometrica *cum* Expreſſione Algebraica *pari ſemper paſſu incedat.*

Sic in exemplo priore, ſi pro *y* ponas *ordinatim*, pro *a* Latus Rectum *Parabolæ*; erit tum *x* abſciſſa Diametri, ordinatæ conveniens. Et ex naturâ hujus Figuræ, quamcunque ſumis longitudinem pro quantitate *y*, 3^tiam proportionalem *x* intra ipſius Schematis metas, potis eris aſſignare in æternum.

4to *Notandum, quod Radicum Impoſſibilium Numerus ſemper ſit par.*

Numerus enim Radicum inæqualium, quæ ſimul deveniunt *æquales*, ſemper fit *par*. Adeoque *parem* etiam oportet eſſe Numerum earum, quæ poſt æqualitatem ſimul *Impoſſibiles* evadunt. Qua vero ratione duæ radices inæquales, ſimul evadunt æquales, hoc eſt, duæ interſectiones curvæ cujuſdam, ſimul convertuntur in punctum contactus, Tyro ex *Geometria* edocendus eſt.

5to Ac propterea, *Imparium Dimenſionum æquationes, omnes ſuas radices Impoſſibiles habere non poſſunt.*

Hiſce jam explicatis, ad Problemata, ſive Praxes neceſſarias, in tractandis æquationibus, progredimur. *Probl.*

Probl. 1. *Quot ſint Radices Impoſſibiles, in æquatione propoſita, invenire.*

Varias hujus regulæ partes prout ordine ſequuntur, in ipsâ praxi trademus; ſic enim tota captu facilior evadet, & menti altius infixa hærebit.

Sit æquatio $x^3 - 4xx + 4x - 6 = 0$, in quâ *nullus omnino terminus deficit*, & ad cujus normam cæteræ omnes (quotcumque ſint dimenſionum) *nullo vel uno ſolum termino carentes*, tractandæ ſunt.

Scribe numeros naturales uſque ad numerum dimenſionum æquationis propoſitæ qui hîc eſt, 3, *viz.* 1. 2. 3. Et his, eoſdem numeros ſuperſcribe, ſed tamen ordine inverſo, qua ratione prodeunt Fractiones ſequentes $\frac{1}{1}$. $\frac{2}{2}$. $\frac{3}{3}$. Divide poſteriorem unamquamque per priorem, *viz.* $\frac{1}{1}$) $\frac{2}{2}$ ($\frac{1}{1}$, & $\frac{2}{2}$) $\frac{3}{3}$ ($\frac{1}{1}$. Ortos Quotientes $\frac{1}{1}$, & $\frac{1}{1}$, collocа ſuper terminis mediis æquationis, hoc modo, $x^3 - 4^{\frac{1}{1}}xx + 4^{\frac{1}{1}}x - 6 = 0$. Sumantur Quadrata terminorum mediorum, ducta in fractiones, eorum capitibus imminentes; *viz.* $\frac{16x^4}{3}$, & $\frac{16xx}{3}$. Comparentur hæc cum Rectangulis terminorum utrinque conſiſtentium, *viz.* $\frac{16x^4}{3}$ cum $x^3 \times 4x$, & $\frac{16xx}{3}$ cum $4xx \times 6$. Prout hæc Rectangula *majora* ſunt, vel *non majora* quantitatibus prædictis, quibuſcum comparantur, ſic ſub ter-

minis *mediis correſpondentibus* ſcribe — vel +: ut quoniam $4x^4 < \frac{16x^4}{3}$, & $24xx > \frac{16xx}{3}$, ergo ſub termino $4xx$ ſcribo +, & — ſub $4x$. Ac tum ſignis ſuis diſtincta, ſic apparet æquatio, *viz.* $x^3 - 4^{\frac{1}{3}}xx + 4^{\frac{1}{3}}x - 6 = 0$. Sub primo & ultimo

\+ —

æquationis termino, in omni caſu colloca ſignum +, adeo ut jam totus ſignorum *ſubſcriptorum* ordo ſic ſe habeat, *viz.* $x^3 - 4^{\frac{1}{3}}xx + 4^{\frac{1}{3}}x - 6 = 0$.

\+ + — +

Hiſce ſic peractis, tenenda eſt hæc Regula, *Tot ſunt radices Impoſſibiles, quot ſunt horum ſignorum (qua infra ponuntur) variationes* de + in — & — in +. Ac propterea in hac æquatione duæ ſunt hujuſmodi radices, quoniam duæ contingunt ſignorum variationes.

Eodem modo in æquatione $x^5 - 4x^4 + 4x^3 - 2xx - 5x - 4 = 0$, duæ Impoſſibiles detegentur.

Eſt enim $\frac{32}{5}x^5 > 4x^5$, & $8x^6 = 8x^6$, & $2x^5 > -20x^4$, & $10xx > 8xx$. Ac propterea ſigna hoc ordine ſubſcribuntur, *viz.* + + — + — +.

Ubi Terminus aliquis (ſed non niſi unus) *deficit, concipere debemus cyphram in ejus loco*, & in cæteris procedere ut ſupra, eodem plane modo ac ſi terminus ille non abeſſet.

Ex. gr. Sit æquatio $x^4 + 6xx - 3x - 2 = 0$, Ergo Numerorum naturalium ſeries 1. 2. 3. 4.

Et Fractiones requiſitæ $\frac{4}{1}$. $\frac{4}{2}$. $\frac{4}{3}$. $\frac{4}{4}$.

Et orti Quotientes $\frac{4}{1}$) $\frac{1}{2}$ ($\frac{1}{8 \cdot 2}$) $\frac{2}{3}$ ($\frac{4}{9 \cdot 3}$) $\frac{1}{4}$ ($\frac{1}{8}$.

Æqua-

Æquatio cum Fractionibus ſuprafcriptis. $x^{4\frac{1}{8}} * - 6xx^{\frac{4}{9}} - 3x^{\frac{1}{8}} - 2 = 0.$

Quadrata Terminorum mediorum in Fractiones ducta
$$0 \times \tfrac{1}{8} = 0$$
$$\frac{144x^4}{9}$$
$$\frac{27xx}{8}$$

Rectangula Terminorum utrinque conſiſtentium circa terminos medios.
$$-6x^6$$
$$0 \times 3x = 0$$
$$12xx$$

Comparatio Rectangulorum cum Quadratis in Fractiones ductis.
$$\text{eſt } -6x^6 < 0$$
$$\& \quad 0 \times 3x < \frac{144x^4}{9}$$
$$\& \quad 12xx > \frac{27xx}{8}$$

Adeo ut ſub * collocandum eſt +, ſub $6xx$ etiam +, & ſub $3x$ ſignum —.

Unde ponendo etiam + ſub primo & ultimo termino, ſignorum ſubſcriptorum talis ordo prodit, $x^{4\frac{1}{8}} * - 6xx^{\frac{4}{9}} - 3x^{\frac{1}{8}} - 2 = 0$, ex quo conclu-
$\quad + \quad + \quad + \quad - \quad +$

ditur æquationem propoſitam duas radices Impoſſibiles continere.

Quando *duo vel plures termini ſimul deficiunt*, Praxis inſtituenda eſt, methodo in paucis quibuſdam a priore diverſa. Sub *Primo* nempe Terminorum deficientium ſcribe ſignum —, ſub *Secundo* +, ſub *Tertio* —, & ſic deinceps, alternatim mutando ſigna: Niſi *ſigna terminorum (qui circa deficientes proxime conſiſtunt) contraria* fuerint, tunc enim ſub *ultimo* eorum qui deſunt, ſignum + ſemper collocandum eſt.

Ut in æquatione $x^5+ax^4+a^5=0$, vel $x^5+ax^4 * * *+a^5=0$, ſub x^5 & a^5 *primo* & *ultimo æquationis* termino, ſcribi debet +, ſub ax^4 etiam +, propterea quod $\frac{4}{5}aax^3 > x^5 \times 0$, & (ſecundum regulæ præſcriptum) ſub deficientium Primo termino collocandum eſt —, ſub *ſecundo* +, ſub *tertio* —, quoniam terminorum ax^4 & a^5 ſigna non ſunt contraria. Adeo ut jam facta imperatâ ſignorum diſpoſitione, æquatio ſic apparet, $x_5+ax^4 * * *+a^5=0$: in quâ quatuor ſunt ſignorum variationes, & per conſequens quatuor radices impoſſibiles. Si vero terminorum ax^4 & a^5, ſigna fuiſſent contraria, ut in hâc æquatione $x^5+ax^4 * * * -+a^5=0$, tunc ultimus terminorum deficientium, eam ob cauſam ſignum + ſub ſe ſcriptum habuiſſet, ac tum duæ tantum fuiſſent Impoſſibiles radices. Quod idem etiam accidit in æquatione ſequenti, quam Tyronum in gratiam paulo uberius tractabimus; ubi tamen alias ſimul ob cauſas fit, ut ſex Impoſſibiles reperiantur. Sit nempe $x^7-2x^6+3x^5-2x^4+x^2 * *-3=0$.

Nume-

Numerorum series pro denominatoribus

Fractionum	1.	2.	3.	4.	5.	6.	7.
Et ipſæ Fractiones	$\frac{1}{1}$.	$\frac{2}{2}$.	$\frac{1}{3}$.	$\frac{2}{4}$.	$\frac{1}{5}$.	$\frac{2}{6}$.	$\frac{1}{7}$.
Orti quotientes	$\frac{1}{7}$.	$\frac{1}{9}$.	$\frac{1}{5}$.	$\frac{1}{5}$.	$\frac{1}{9}$.	$\frac{1}{7}$.	

Æquatio cum Fractionibus adſcriptis

$$x^7 \overset{\frac{1}{7}.}{-} 2x^6 \overset{\frac{1}{9}.}{+} 3x^5 \overset{\frac{1}{5}.}{-} 2x^4 \overset{\frac{1}{5}.}{+} x^3 \overset{\frac{1}{9}.}{*} \overset{\frac{1}{7}.}{*} - 3 = 0$$

Quadrata terminorum mediorum ducta in Fractiones.

$$\frac{12}{7}x^{12}. \quad \frac{49}{9}x^{10}. \quad \frac{12}{5}x^8. \quad \frac{3}{5}x^6$$

Rectangula terminorum qui medios proximè includunt.

$$3x^{12}. \; 4x^{10}. \; 3x^8. \; 0.$$

Comparatio Quadratorum ductorum in fractiones, cum Rectangulis.

$$\frac{12}{7} < 3. \frac{49}{9} > 4.$$

$$\frac{12}{5} < 3. \frac{3}{5} > 0.$$

Propterea ſubtermino 2*do aequationis* oportet ſcribere ſignum —, ſub 3*tio* +, ſub 4*to* —, ſub 5*to* +, ſub primo termino *deficientium* —, ſub 2*do* (qui etiam eſt ultimus eorum) ſcribendum eſt +, ſigna enim terminorum x^1 & 3 ſunt *contraria*; & tandem ſub

primo & ultimo æquationis termino, ſignum + uti ſæpius monitum fuit. Signorum ergo diſpoſitio talis eſt, *viz.*

$$x^7 - 2x^6 + 3x^5 - 2x^4 + x^3 * * - 3 = 0,$$
$$+ \quad - \quad + \quad - \quad + - +$$

quâ indicatur, unam tantum eſſe *realem*, ſex vero *Impoſſibiles* radices hujus æquationis.

Schol. 1. Notandum vero eſt, *plures eſſe poſſe radices Impoſſibiles quam per hanc regulam in conſpectum veniunt*; ſed hoc *perraro contingit*, ita Judicante & pronunciante *Celeb. Newtono*, cui regulas haſce debemus.

Schol. 2. Etſi *plures eſſe poſſunt* radices Impoſſibiles, quam quæ hujus Regulæ auxilio deteguntur; ſupponit tamen illa, *tot eſſe ejuſmodi Radices, quot ſunt ſignorum ſubſcriptorum variationes*: Et ergo, ex hoc Symptomate (ubicunque apparet) radicis Impoſſibilis præſentiam colligit. *In æquationibus ergo, quarum omnes radices ſunt* poſſibiles *aut* reales, *nulla unquam ſignorum ſubſcriptorum variatio contingere poterit.* Fieri quidem poteſt *ut non contingat variatio*, ubi tamen *latet radix Impoſſibilis* (quoniam plures eſſe poſſunt, quam quas regula in lucem profert, & regula calculum inire profitetur ex ſignorum ſubſcriptorum variationibus) ſed ubi contingit variatio, *ibi radicem impoſſibilem non inveniri*, a ſenſu ac mente regulæ plane alienum eſt

Adeoque in æquationibus, quæ non niſi *reales* radices involvunt, *nulla unquam ſignorum ſubſcriptorum mutatio expectanda eſt.*

Sed

Sed neceſſario orietur ſignorum variatio, ſi collocato ſigno + ſub *primo* & *ultimo* termino, ſigna omnium terminorum intermediorum non fuerint ſimiliter *affirmativa.* Et ſecundum regulam ſigna terminorum mediorum affirmativa haud quaquam fuerint, *niſi eorum Quadrata, ducta in Fractiones imminentes, Rectangula ſub terminis utrinque proximis perpetuo ſuperent.* Quapropter in æquationibus quarum *omnes radices reales* ſunt, *Quadrata* terminorum mediorum ducta in fractiones imminentes, *Rectangula* ſub terminis utrinque proximis, *perpetuo ſuperare debent.* Igitur hoc univerſaliter demonſtrato, *ex Legibus Coefficientium quibus medii æquationis termini affecti ſunt,* manifeſta erit ratio hujus regulæ, quæ ex cauſâ contrariâ (nimirum Quadratorum ductorum in fractiones defectu a Rectangulis) radices Impoſſibiles colligere docet. Sufficit vero (ad hoc *illuſtrandum)* exemplum in numeris adjicere.

Sit æquatio $z^3-8z^2+17z-10=0$, cujus radices ſunt 5. 2. 1. omnes *reales* & *affirmativæ.* Erit Jam $64\times\frac{1}{3}>17$ & $289\times\frac{1}{3}>80$: adeo ut ſub termino 2*do* & 3*tio* ſignum + ſcribendum eſſet. Si radices eſſent 1. 2. — 5, unde æquatio $x^3+2x^2-13x+10=0$; hoc in caſu, eſt $\frac{4}{3}>-13$, & $\frac{169}{3}>20$, ac propterea ſignum +, iterum collocandum ſub 2*do* & tertio termino. Quod idem eveniret, ſi radices eſſent —1. —2. +5, & æquatio $x^3-2x^2-13x-10=0$; vel ipſæ —1. —2. —5, & æquatio $x^3+8x^2+17x+10=0$: ut ipſo oculi intuitu patet.

Sic ſumptâ quâvis aliâ, quarumcunque dimenſionum æquatione (cujus omnes radices ſunt reales) idem experiri liceat.

Probl.

Probl. 2. *An Radices Impoſſibiles, inter Affirmativas vel Negativas lateant dignoſcere.*

Obſerventur ſigna terminorum in æquatione, ſignis *ſubſcriptis variantibus*, immineatium. Tot enim *Affirmativæ* ſunt Impoſſibiles *quot ſunt horum ſignorum variationes*, & tot negativæ Impoſſibiles, *quot ipſorum ſucceſſiones ſine variatione.*

Nam ex *æquationum Generatione*, tot ſunt radices *Reales*, affirmativæ vel negativæ, quot ſunt variationes vel ſucceſſiones ſignorum *in æquatione.* Et ex *regula allata*, tot ſunt *Impoſſibiles*, quot ſignorum *infra ſcriptorum* variationes.

Ergo prout variationes ſignorum *ſubſcriptorum* reſpondent vel variationibus vel ſucceſſionibus ſignorum *ſuprаſcriptorum*, ita oritur indicium radices Impoſſibiles, vel inter affirmativas vel negativas latere.

Ut in æquatione $x^4 - 4x^3 + 4x^2 - 2xx - 5x - 4$
$+ \quad + \quad - \quad + \quad + \quad +$
$= 0$, ſigna *ſubſcripta* variantia ſunt
$+ \quad - \quad +$, quibus ſuperius reſpondent
$- \quad + \quad -$, nempe terminorum $-4x^3$, $+4x^2$ & $-2xx$.

Jam duæ ſignorum ſubſcriptorum variationes, duas *Impoſſibiles* denotant, & duæ ſignorum ſuprаſcriptorum variationes indicant duas radices *Affirmativas*: ac per conſequens latet Impoſſibilitas inter affirmativas, ſive duæ prædictæ affirmativæ ſunt Impoſſibiles. Adeoque cum ſigna æquationis propoſitæ (hoc ordine ſe habentia) $+ - + - - -$, indicant tres affirmativas, duaſque negativas, & duæ ex affirmativis impoſſibiles ſunt;

ſequitur

sequitur radices æquationis sic computandas esse, ut *una* tantum sit vera vel *realis affirmativa*, *duæ Impossibiles* etiam affirmativæ, & *duæ Negativæ reales.*

Probl. 3. *Æquationis cujusvis radices, data quævis quantitate, augere vel minuere.*

Hoc est datâ quâlibet æquatione, aliam formare, in quâ radicum valores, sint datâ quantitate majores vel minores, valoribus radicum æquationis propositæ.

Ponatur alia incognita + *vel* — *quantitate data, æqualis radici æquationis datæ, in cujus locum per totam æquationem nova hinc orta expressio subroganda est.*

Ex. gr. Sit æquatio $y^4+3y^3-15y^2-19y+30=0$, & sit radix y *quaternario* augenda.

Ponatur $z=y+4 \therefore z-4=y$, unde

$$\begin{array}{rrrrr} z^4 & -16z^3 & +96z^2 & -256z & +256=y^4 \\ & +3z^3 & -36z^2 & +144z & -192=3y^3 \\ & & -15z^2 & +120z & -240=-15y^2 \\ & & & -19z & +76=-19y \\ & & & & +30 \end{array}$$

Summa $z^4-13z^3+45z^2-11z-70=0$; ubi radices (quæ prius erant, 1, 3, —2, —5,) jam quaternario auctæ, evadunt 5, 8, 2, — 1, unâ scilicet ex negativis in affirmativam conversâ. Sed ut hujus problematis natura, tum & mutationum inter radices inde fluentium ratio, melius percipiantur, formetur æquatio ex quibuscunque

cunque radicibus, *viz.* $y-a=0$, $y+b=0$, ita ut

$$\begin{matrix} y^2-a \\ +b \end{matrix} \; y-ab=0,$$ sit æquatio, cujus ra-

dix datâ quantitate n, ex. gr. Sit augenda. Fiet ergo $z=y+n$, $\therefore z-n=y$, & substituendo oritur

$$\begin{matrix} -2n \\ zz-a \\ +b \end{matrix} \; z+nn+na-nb-ab=0.$$

Hæc vero æquatio produceretur ex multiplicatione $z-a-n$ in $z+b-n$, adeo ut ipsius z valores sunt $a+n$, & $-b+n$; ac per consequens habetur nova æquatio, in qua singulæ radices majores sunt quam illæ prioris, datâ quantitate n.

Affirmativa quæ prius fuit a, Jam evadit $a+n$, & Negativa $-b$ etiam fit $-b+n$, quæ quidem *major* est quam $-b$, per n, adhuc vero manet *negativa*; supposito quod b ipsam n excedat. Expressio $-b+n$, *major est quantitas* quam $-b$, etiam si per *minorem numerum* eam explicari necesse sit. Uti $-4+3$ vel -1, est quid majus quam -4, attamen 1 est minor numerus quam 4. Unde *Stylo Cartesiano* diceremus, *radicem falsam* (sic enim loqui amant) *eadem quantitate minui, qua vera augetur.*

Si ponamus $n=b$, tunc æquatio convertitur in hanc, $zz-az-nz=0$, vel $z-a-n=0$. Hoc est, *si augeatur Radix affirmativa, quantitate alicui Negativæ æquali, Evanescet illa negativa, & per consequens æquationis dimensiones uno gradu minuentur.*

Si

Si $n > b$, tunc $-a+b-2n$ signo negativo, & $nn-nb+an-ab$ signo affirmativo gaudebit, adeo ut radix negativa hoc pacto affirmativa evadat. Hoc est, *si Radix affirmativa augeatur quantitate majore quadam ex negativis, illa negativa in affirmativam convertetur: & per consequens si radicis affirmativæ incrementum maximam radicum negativarum excedat, omnes radices negativæ in affirmativas transmutabuntur.*

Si $n < b$, tunc quantitas $nn-nb+an-ab$ semper Negativa erit, sed $-a+b-2n$ Affirmativa, si $b > a+2n$, negativa vero si $b < a+2n$: *Sed Casu utrovis, una radix erit affirmativa, & altera negativa, etiamsi in priore, negativa sit major, at in posteriore, affirmativa.*

Ex paucis hisce, facile colligi potest quid contingere debet, si radix *affirmativa minuenda* esset, vel è contra *Negativa augenda*, quantitate aliquâ datâ; idque etiam in quibuscumque sublimiorum dimensionum æquationibus: ut opus non sit cur harum rerum explicationi diutius insistamus.

Problema hoc, *usibus sequentibus inservire potest.*

Primo, *Æquationis cujuslibet datæ, terminum secundum auferre*, vel potius eam in aliam æquationem mutare, quæ termino 2*do* careat.

Sit $x^3-lxx+mmx-n^3=0$, ponaturque $z=x\pm q$; *signum* enim, non minus quam ipsa quantitas q, investigari debet, & ideò tanquam indeterminatum relinqui. Est ergo $z\mp q=x$, quo valore

Ut in prædictâ æquatione $x^3 - lxx + mmx - n_3 = 0$, ubi ponendo $x = z + q$, resultat

$$\begin{array}{r} z^3 + 3zzq + 3zq^2 + q^3 \\ -lzz \quad -2lzq \quad -lq^2 \\ +m^2z + m^2q \\ -n^3 \end{array} = 0.$$

Ut evanescat terminus 3*tius*, debet esse $3q^2 - 2lq + m^2 = 0$, hoc est $q^2 - \frac{2}{3}lq = -\frac{m^2}{3}$ vel $q^2 - \frac{2}{3}lq + \frac{1}{9}ll = \frac{1}{9}ll - \frac{mm}{3}$, adeoque $q = \frac{1}{3}l \pm \sqrt{\frac{1}{9}ll - \frac{mm}{3}}$. Hinc cum $z + q$ scripta sit pro x, scribi etiam debet $z + \frac{1}{3}l \pm \sqrt{\frac{1}{9}ll - \frac{mm}{3}}$ pro x in æquatione propositâ, ut oriatur alia æquatio, in quâ deficiat *Tertius*, adsit vero *secundus*, Terminus: Quod idem in Numeris illustrari facile poterit.

Aliorum Terminorum (puta 4*ti*, 5*ti*, &c.) sublatio, altiorum adhuc æquationum resolutionem postularet; quæ ergo ut labor inutilis merito prætereunda est.

Secundo, ope hujus artificii, quo mutantur (augentur nempe vel minuuntur) *valores radicum; æquationes deficientes compleri possunt, adeo ut nullus omnino terminus absit:* Et præterea, ut æquatio ad datum quemlibet numerum dimensionum *assurgat.*

Ex. gr. Si sit $x^3 * - rx + q = 0$, pone $x = y - a$, & hujus potestates loco potestatum ipsius x quæ inveniuntur in æquatione propositâ; & orietur *Cubica*, in qua nullus terminus deficiet. Si desideretur

deretur *Biquadratica* completa, multiplicentur ſinguli termini æquationis datæ per ipſam x, unde fit $x^5 * - rx^2 + qx = 0$, & tum ubique ſubſtituatur $y - a$ pro x. Sic ſi eſſet $x^7 * * * * - bxx + cx - r = 0$, & requiratur ejus loco, alia *octo* dimenſionum æquatio, omnibus ſuis terminis gaudens; ſcribatur $x^8 * * * * * - bx^3 + cx^2 - rx = 0$, & ſupponendo $x = y - m$, fabricetur *æquatio nova* ex hoc valore radicis x, prout poteſtates ipſius æquationis incompletæ dirigunt.

Tertio, utilis eſt hæc operatio ad explorandas radices Impoſſibiles, quas regula prædens tectas at telatas interdum relinquit.

Uti ſi $z^3 - 3ddz - 3d^3 = 0$, tentando radices Impoſſibiles ſecundum regulæ præſcriptum, nullæ omnino invenientur. Scriptis enim (ſuper terminis mediis) fractionibus æquationis naturæ convenientibus, $z^3 * ^{\frac{1}{3}} - 3d^{\frac{1}{3}}dz - 3d^3 = 0$, erit $0 > -3dd$, & $3d^4 > 0 * - 3d^3$ $(= 0)$ unde ſigna ſubſcripta hoc ordine ſtant, $+ + + +$; adeoq; nulla concludi poterit radix impoſſibilis.

Augeatur vero radix z, ut ſit *ex. gr.* $y - q = z$, ergo $y^3 - 3y^2q + 3yq^2 - q^3 - 3yd^2 + 3d^2q = 0$. Jam liquet quod $\frac{1}{3} \times 9q^2$ vel $3q^2 > 3q^2 - 3d^2$, & ergo ſub termino $2do$ ſcribi debet ſignum $+$. Sed comparando $\frac{9q^4 - 18q^2d^2 + 9d^4}{3}$ cum quantitate $3q^4 - 9q^2d^2 + 9d^4q$ (hoc eſt tertiam partem *Quadrati* ter-

mini medii cum *Rectangulo* terminorum utrinque conſiſtentium) vel $9q^4 - 18q^2d^2 + 9d^4$ cum $9q^4 - 27q^2d^2 + 27d^3q$, vel (*abjiciendo* utrinque $9q^4$, *addendo* $18q^2d^2$, & tandem *dividendo* per $9d^2$) comparando d cum $3dq - q^2$; ſi $d^2 > 3dq - qq$, tum ſub 3*tio* termino etiam ſcribi debet ſignum $+$, nullaque adeo erit radix Impoſſibilis. Si vero ſit $d^2 < 3dq - qq$, eo in caſu ſignum $-$ prædicto termino ſubſcribere oportebit, & tum ſtatim apparebunt duæ radices Impoſſibiles.

Ergo manifeſtum eſt, quomodo radicis *Incrementum ſumi debet*, adhoc ut radices Impoſſibiles *prius latentes, jam ſe prodant.* Et quidem ſi fiat $q = d$, æquatio in hanc formam ſe convertet, $y^3 - 3y^2q * - d^3 = 0$, ubi etiam primo aſpectu duæ impoſſibiles deteguntur.

Prob. 4. *Æquationis data radices, per datam quamvis quantitatem, multiplicare vel dividere.*

Supponatur alia quævis incognita, *multipla* vel *ſubmultipla* (*in ratione data*) radicis æquationis propoſitæ, cujus radicis, hinc ortus valor novus, in ejus locum ubique ſubrogandus eſt, & ſic conflabitur æquatio quæſita.

Sit $z^3 - azz + nnz - p^3 = 0$, & ponatur $v = rz$, ergo $z = \frac{v}{r}$, unde fiet $\frac{v^3}{r^3} - \frac{av^2}{r^2} + \frac{nnv}{r} - p^3 = 0$; hoc eſt ductis omnibus in r^3, $v^3 - rav^2 + rrnnv - p^3r^3 = 0$.

Si exponatur quantitas r per numerum quemlibet *Integrum* operatio hæc, eſt *Multiplicatio* radicis

dicis (z) per eundem numerum: Si vero r ſit numerus *Fractus*, tunc ſit diviſio ejuſdem radicis z per eundem. Hoc eſt, radix v eſt ipſius z, *multipla* vel *ſubmultipla*, in ratione datâ r ad 1: *Ex. gr.* ſi $r=3$, ſive $z=\frac{v}{3}$, erit $\frac{v^3}{27}-\frac{avv}{9}+\frac{nnv}{3}-p^3=0$, vel omnibus multiplicatis per 27, $v^3-3avv+9nnv-27b^3=0$. Si $r=\frac{1}{3}$, vel $z=3v$, erit $27v^3-9avv+3nnv-p^3=0$, hoc eſt diviſis omnibus per 27, $v^3-\frac{avv}{3}+\frac{nnv}{9}-\frac{p^3}{27}=0$.

Ex quibus manifeſtum eſt, *quod quantitates cognitæ in nova æquatione, ſint quantitates cognitæ æquationis datæ* reſpective ductæ *in poteſtates ipſius quantitatis, per quam radix multiplicanda vel dividenda eſt.*

Hæc operatio uſui erit———

Primò, *ad faciendam quantitatem cognitam alicujus termini in æquatione, æqualem alicui alteri datæ quantitati quam quis ſibi fingere velit.*

Ut ſi loco prioris æquationis $z^3-azz+nnz-p^3=0$, aliam poſtularem, in quantitas cognita 2*di* termini non eſſet *a* ſed *d*. Ponatur $v=zq$, vel $z=\frac{v}{q}$; ubi oportet determinare valorem quantitatis q in quantitatibus datis *a* & *d*. Quapropter ſubſtituendo $\frac{v}{q}$ in locum ipſius z, erit $\frac{v^3}{q^3}-\frac{av^2}{q^2}+\frac{nnv}{q}-p^3=0$, vel $v^3-aqv^2+nnqqv-q^3$

$p^1=0$. Jam ex *Hypotheſi* quantitas cognita 2di termini debet eſſe d, ergo $aq=d$, vel $q=\frac{d}{a}$, & ponendo $\frac{d}{a}$ pro q in æquatione ultimâ, erit $v^3-dv^2+\frac{nndd}{aa}v-\frac{p^3d^3}{a^3}=0$, æquatio in quâ quantitas cognita 2di termini eſt d, prout *imperatum* eſt.

Si loco prioris alia requireretur æquatio, in quâ tertius terminus non eſſet nn ſed ff; tunc factâ (ut priùs) novâ æquatione, oportet ut ſit $nnqq=ff$, vel $q=\frac{f}{n}$, unde reſultabit æquatio $v^3-\frac{af}{n}v^2+ffv-\frac{f^3p^3}{n^3}=0$. in quâ quantitas cognita 3tii termini eſt f^2.

Secundo, Hac ratione æquationes (quandoque) liberari poſſunt ex Terminis Surdis & Fractis, adeoque ad expreſſiones pure Rationales & Integras perduci.

Sit æquatio $x^3-x^2\sqrt{p}+a^2x-\frac{d}{\sqrt{p}}=0$, & fiat $z=xq$, unde $\frac{z}{q}=x$, ergo $\frac{z^3}{q^3}-\frac{z^2\sqrt{p}}{q^2}+\frac{a^2z}{q}-\frac{d}{\sqrt{p}}=0$, & $z^3-qz^2\sqrt{p}+q^3a^2z-\frac{q^3d}{\sqrt{p}}=0$.

Si

Si jam quantitas indeterminata q esset $=\sqrt{p}$ vel $p\sqrt{p}$ vel $p^2\sqrt{p}$ vel $p^3\sqrt{p}$ vel $p^4\sqrt{p}$, *&c.* hoc est $p^{\frac{1}{2}}$, $p^{\frac{1}{2}}$, $p^{\frac{1}{2}}$, $p^{\frac{2}{2}}$, $p^{\frac{2}{2}}$, &c. Tunc *omnes termini rationales essent*, hoc est $\sqrt{p}$ non amplius in æquatione appareret. Sed hoc non eveniret, si q exponenda esset per p. p^2. p^3. &c. etenim termini $q\sqrt{p}$ & $\frac{q^3}{\sqrt{p}}$ semper manerent irrationales. Fiat ergo $q=\sqrt{p}$, ut pote *simplicissima* expressioni, tum erit, $z^3 - q^2 z^2 + q^2 a^2 z - q^3 d = 0$, æquatio desiderata. Ubi advertere oportet quod (quantitas cognita 2*di* termini) $\sqrt{p}$ multiplicatur per q, & a^2 (quant. cogn. 3*tii*) per q^2, & $\frac{d}{\sqrt{p}}$ (terminus 4*tus*) per q^3: Est enim $q^2 = \sqrt{p} \times q$, $q^2 a^2 = a^2 \times q^2$, $q^3 d = \frac{d}{\sqrt{p}} \times q$.

In *numeris*, Sit $x^3 - x^2\sqrt{3} + \frac{26}{27}x - \frac{8}{27\sqrt{3}} = 0$. Hic invenitur $a^2 = \frac{26}{27}$, $d = \frac{8}{27}$, $\sqrt{p}$ vel $q = \sqrt{3}$, quibus valoribus substitutis in æquatione priore, oritur $z^3 - 3zz + \frac{26}{9}z - \frac{8}{9} = 0$.

Proponatur ulterius æquatio in terminis generalibus $z^3 - mzz + \frac{az}{d} - \frac{b}{d} = 0$, & sit ea *a fractionibus liberanda*. Concipiatur $v = rz$, & investigetur quantitas r in hunc finem necessaria, sicuti prius quærebatur quantitas q pro *tollenda Assymetria*. Propterea ubique scribendo $\frac{v}{r}$ loco

ipſius z, erit (facta multiplicatione per r^3) $v^3 - mrv^2 + \frac{arrv}{d} - \frac{hr^3}{d} = 0$. Hiſce poſitis permultæ fieri poſſunt ſuppoſitiones, in quibus omnibus *Fractiones penitus evaneſcent*. Ut ſi ſtatuatur quantitas r æqualis *alicui multiplo* quantitatis d, ipſis d & m interim *numeris integris* exiſtentibus. Vel etiam ſi r vel $rr = d$, & m ſit quilibet numerus *integer*. Vel denique ſi $r = m = \sqrt{d}$, & m per numerum integrum exponatur.

Omnes verò modos quibus hoc peragere quis poſſit, enumerare non ſuſcipio. Si contingat ex. gr. ut ſit $m^3 = d$, r fiat $= m$, & habebitur æquatio $v^3 - mmv^2 + av - hm = 0$, *ab omni Fractione libera*. In *numeris*, ſi $z^3 - 3zz + \frac{26}{9}z - \frac{8}{9} = 0$, ubi $3 = m$, $9 = d$, $26 = a$, $8 = h$; ponatur $v = 3z$, & tandem reſultabit $v^3 - 9vv + 26v - 24 = 0$. Ubi iterum apparet multiplicatio quantitatum cognitarum datæ æquationis reſpectivè per terminos ſerici continuè proportionalium.

Schol. Ex operationibus in hoc Problemata explicatas videre eſt, quomodo æquationes quarum radices *Quantitates ſurdas vel fractas involvunt*, vel per Numeros rationales & integros, forſan vero *magnos & difficiles* exprimuntur, in alias faciliores tranſmutandæ ſint, quarum radices ſint *rationales* vel *integræ*, vel datâ quâlibet quantitate minores quam *prius*.

Prob. 5. *Æquationis cujuſlibet radices, in earum Reciprocas tranſmutare*; hoc eſt æquationem formare, cujus radices ſint Reciprocæ radicum datæ æquationis.

Sit

Sit æquatio $x^3 - 2x^2 - 23x + 60 = 0$, & fiat $z = \frac{1}{x}$, $\therefore x = \frac{1}{z}$, unde $\frac{1}{z^3} - \frac{2}{z^2} - \frac{23}{z} + 60 = 0$, & $1 - 2z - 23zz + 60z^3 = 0$, vel $z^3 - \frac{2}{60}zz - \frac{1}{30}z + \frac{1}{60} = 0$. *Datæ* æquationis radices fuerunt 4, 3, -5, ergo æquationis novæ radices sunt $\frac{1}{4}$, $\frac{1}{3}$, $-\frac{1}{5}$, nempe priorum reciprocæ: quod per se liquet, producetur enim hæc æquatio ex multiplicatione $z - \frac{1}{4} = 0$, $z - \frac{1}{3} = 0$, $z + \frac{1}{5} = 0$, in se invicem. Adeo ut hoc pacto radix *maxima* convertatur in *minimem*, & *minima* in *maximam*; & in hoc exemplo, radix 3 in $\frac{1}{3}$ & 4 in $\frac{1}{4}$, & -5, jam facta $-\frac{1}{5}$ nihilo propius accedit. Per hanc operationem.

Primò, *Æquationes termino secundo Carentes sed tertio gaudentes, transmutantur in alias, quæ quidem terminum secundum habeant, sed tertio careant.*

Ut si $x^3 * + 3m^2x - n^3 = 0$, fiat $x = \frac{1}{z}$, unde $z^3 + \frac{3m^2}{z} - n^3 = 0$, & $1 + 3m^2z^2 - n^3z^3 = 0$, vel $z^3 - \frac{3m^2}{n^3}z^2 * - \frac{1}{n^3} = 0$.

Secundo, viciſſim quæ æquationes termino ſecundo gaudent ſed tertio carent, hâc ratione in alias convertuntur, quæ ſecundo careant & tertio gaudeant.

Quod manifeſtum eſt ex ultimâ æquatione $z^3 - \frac{3m^2}{n^3}z^2 * - \frac{1}{n^3}$, quæ (ſi ponatur $z = \frac{1}{x}$) erit $\frac{1}{x^3} - \frac{3m^2}{n^3x^2} - \frac{1}{n^3} = 0$, hoc eſt $1 - \frac{3m^2x}{n^3} - \frac{x^3}{n^3} = 0$, vel $x^3 * + 3m^2x - n^3 = 0$.

Sic etiam & *Bi-quadratica*, in quibus inveniuntur termini 3*tius*, & 4*tus*, non vero 2*dus*, in alias mutantur, quæ terminos 2*dum* & 3*tium*, non vero 4*tum* habeant. Ut si $x^4 * - px^2 + qx - r = 0$, fiat $x = \frac{1}{z}$, quare $\frac{1}{z^4} - \frac{p}{z^2} + \frac{q}{z} - r = 0$, hoc est $1 - pz^2 + qz^3 - rz^4 = 0$, vel $z^4 - \frac{q}{r} z^3 + \frac{p}{q} z^2 * - \frac{1}{r} = 0$.

Tertio, operatione haud multum dissimili æquationes deficientes compleri poterunt. Revocetur enim æquatio $x^4 * - px^2 + qx - r = 0$, & ponetur $x = \frac{1}{z^{\frac{1}{2}}}$, quare $\frac{1}{zz} - \frac{p}{z} + \frac{q}{z^{\frac{1}{2}}} - r = 0$, hoc est $1 - pz + qz^{\frac{3}{2}} - rz^2 = 0$, ergo $qz^{\frac{3}{2}} = pz + rz^2 - 1$, unde partem utramque *quadrando*, erit $qqz^3 = ppz^2 + rrz^4 + 2prz^3 - 2rz^2 - 2pz + 1$, quæ est æquatio Bi-quadratica *nullo termino* carens. Idem in aliis præstari poterit, imo & cum *aliis assumptis potestatibus quantitatis insognitæ*, etiamsi *non eadem facilitate.*

Ut si $y^3 * - qy + s = 0$, fiat $y = \frac{1}{z^{\frac{1}{3}}}$, quare $\frac{1}{z} - \frac{q}{z^{\frac{1}{3}}} + s = 0$, adeoque $1 - qz^{\frac{2}{3}} + zs = 0$, unde $1 + zs = qz^{\frac{2}{3}}$, & multiplicando *Cubice* erit $1 + 3zs + 3zzss + z^3s^3 = q^3z^2$.

Probl.

Probl. 6. *Æquationem datam Quantitates irrationales involventem, in puris rationalibus exprimere,* Idque in terminis istius incognitæ quæ primo in æquatione reperitur.

Sit $\sqrt[3]{bbx} - \sqrt{dx} + c = 0$. Pone $\sqrt[3]{bbx} = y$, $\sqrt{dx} = z$, unde fit æquatio sequens $y - z + c = 0$. Jam hîc duæ sunt incognitæ diversæ, ex quibus oportet æquationem componere, quæ *rationales* duntaxat Potestates ipsius x contineat. In hunc finem sumantur duo valores incognitæ x, ex æquationibus suppositiis resultantes, nempe $x = \frac{y^3}{b} = \frac{zz}{d}$, $\therefore y^3 = \frac{bbzz}{d}$, $\therefore \sqrt[3]{\frac{bbzz}{d}} - z + c = 0$, vel $\sqrt[3]{\frac{bbzz}{d}} = z - c$, $\therefore$ multiplicando Cubicè $\frac{bbzz}{d} = z^3 - 3z^2c + 3zc^2 - c^3$, & quoniam $\sqrt{dx} = z$, factâ substitutione erit $bbx = dx\sqrt{dx} - 3dxc + 3c^2\sqrt{dx} - c^3$, hoc est, $\frac{bb + 3dxc + c^3}{dx + 3c^2} = \sqrt{dx}$, & quadrata sumendo utrinque, erit $\overline{bbx + 3dxc + c^3}^2 = dx \times \overline{dx + 3c^2}^2$, æquatio *rationalis* invenienda.

Schol. Si detur æquatio plures terminos irrationales involuens, tunc & plures erunt diversi nominis incognitæ. Sed quoniam tot dabuntur æquationes, quot incognitæ, *hæ gradatim exterminari possunt*, & sic (ad hujus exempli normam) obtineri æquatio ab omni quantitate surdâ libera, nec non in terminis incognitæ primò datæ expressa.

Probl.

$p^1=0$. Jam ex *Hypotheſi* quantitas cognita 2di termini debet eſſe d, ergo $aq=d$, vel $q=\frac{d}{a}$, & ponendo $\frac{d}{a}$ pro q in æquatione ultimâ, erit $v^3-dv^2+\frac{nndd}{aa}v-\frac{p^3d^3}{a^3}=0$, æquatio in quâ quantitas cognita 2di termini eſt d, prout *imperatum* eſt.

Si loco prioris alia requireretur æquatio, in quâ tertius terminus non eſſet nn ſed ff; tunc factâ (ut priùs) novâ æquatione, oportet ut ſit $nnqq=ff$, vel $q=\frac{f}{n}$, unde reſultabit æquatio $v^3-\frac{af}{n}v^2+ffv-\frac{f^3p^3}{n^3}=0$. in quâ quantitas cognita 3tii termini eſt f^2.

Secundo, Hac ratione æquationes (quandoque) liberari poſſunt ex Terminis Surdis & Fractis, adeoque ad expreſſiones pure Rationales & Integras perduci.

Sit æquatio $x^3-x^2\sqrt{p}+a^2x-\frac{d}{\sqrt{p}}=0$, & fiat $z=xq$, unde $\frac{z}{q}=x$, ergo $\frac{z^3}{q^3}-\frac{z^2\sqrt{p}}{q^2}+\frac{a^2z}{q}-\frac{d}{\sqrt{p}}=0$, & $z^3-qz^2\sqrt{p}+q^2a^2z-\frac{q^3d}{\sqrt{p}}=0$.

Si

Si jam quantitas indeterminata q esset $=\sqrt{p}$ vel $p\sqrt{p}$ vel $p^2\sqrt{p}$ vel $p^3\sqrt{p}$ vel $p^4\sqrt{p}$, &c. hoc est $p^{\frac{1}{2}}$, $p^{\frac{1}{2}}$, $p^{\frac{1}{2}}$, $p^{\frac{2}{2}}$, $p^{\frac{2}{2}}$, &c. Tunc *omnes termini rationales essent*, hoc est $\sqrt{p}$ non amplius in æquatione appareret. Sed hoc non eveniret, si q exponenda esset per p. p^2. p^3. &c. etenim termini $q\sqrt{p}$ & $\frac{q^3}{\sqrt{p}}$ semper manerent irrationales. Fiat ergo $q=\sqrt{p}$, ut pote *simplicissima* expressioni, tum erit $z^3 - q^2z^2 + q^3a^2z - q^3d = 0$, æquatio desiderata. Ubi advertere oportet quod (quantitas cognita 2*di* termini) $\sqrt{p}$ multiplicatur per q, & a^2 (quant. cogn. 3*tii*) per q^2, & $\frac{d}{\sqrt{p}}$ (terminus 4*tus*) per q^3: Est enim $q^2=\sqrt{p}\times q$, $q^2a^2=a^2\times q^2$, $q^3d=\frac{d}{\sqrt{p}}\times q$.

In *numeris*, Sit $x^3 - x^2\sqrt{3} + \frac{26}{27}x - \frac{8}{27\sqrt{3}} = 0$. Hic invenitur $a^2=\frac{26}{27}$, $d=\frac{8}{27}$, $\sqrt{p}$ vel $q=\sqrt{3}$, quibus valoribus substitutis in æquatione priore, oritur $z^3 - 3zz + \frac{26}{9}z - \frac{8}{9} = 0$.

Proponatur ulterius æquatio in terminis generalibus $z^3 - mzz + \frac{az}{d} - \frac{h}{d} = 0$, & sit ea *a fractionibus liberanda*. Concipiatur $v=rz$, & investigetur quantitas r in hunc finem necessaria, sicuti prius quærebatur quantitas q pro *tollenda Assymetria*. Propterea ubique scribendo $\frac{v}{r}$ loco

& reducendo $z^6 - n^3z^3 - m^6 = 0$, vel $z^3 = \frac{1}{2}n^3 + \sqrt{\frac{1}{4}n^6 + m^6}$ quæ eſt æquatio Cubica ſimplex, & ſemper poſſibilis.

Manente æquatione ut prius, ſi ponamus $x = \frac{mm - zz}{z}$, orietur etiam ſimplex æquatio Cubica, & ſemper poſſibilis.

At ſi æquatio ſit $x^3 - 3mmx - n^3 = 0$, tum ſupponendo $x = \frac{zz + mm}{z}$, producetur æquatio quæ poſſibilis tantum erit cum quantitas m^6 major non invenitur quam $\frac{1}{4}n^6$, aliter enim extrahenda eſſet radix Quadrati Negativi, ex quo concluderetur æquationis datæ tranſmutationem in ſimplicem Cubicam omnino Impoſſibilem eſſe.

Probl. 9. *Æquationum Biquadraticarum ſolutionem oſtendere; ſive qua ratione ea in duas Quadraticas adfectas dividi poſſint.*

Sicut æquationes *cubicæ*, ſecundo termino carentes primò ad æquationem Quadraticam adfectam, deinde ad ſimplicem Cubicam interdum reducere licet; ſic etiam *Biquadraticæ* quarum ſimiliter abſit terminus ſecundus, mediante æquatione *Cubica* adfecta facile in duas *Quadraticas* adfectas diſtribui poterunt, & ſic earum radices inveniri.

Aſſumatur æquatio quam in hujus regulæ exemplum adhibuit ipſe *Carteſius Geom. Lib.* 3. *Pag.* 79. viz. $x^4 * . pxx . qx . r = 0$; vel eam diſtinguendo per ſigna ad libitum, ſit $x^4 * - px^3 - qx + r = 0$. Dicit ille quod hujus loco ſcribi poſſunt duæ

duæ *Quadratica* $xx+yx+\frac{1}{2}yy-\frac{1}{2}p+\frac{q}{2y}=0$, ac $xx-yx+\frac{1}{2}yy-\frac{1}{2}p-\frac{q}{2y}=0$, nimirum reperto valore quantitatis y, ope hujus æquationis *Cubicæ*

$$y^6-2py^4+pp \\ -4ryy-qq=0.$$

Ut Pateat hujus Regulæ Fundamentum, fingantur duæ æquationes *Quadratica* adfectæ ex quarum multiplicatione emergat æquatio *Biquadratica* ejuſdem formæ cum eâ quæ proponitur. Sint eæ $xx+yx+z=0$, ac $xx-yx+v=0$, ubi terminus yx habet ſignum poſitivum in unâ, ſed negativum in alterâ, eo fine ut evaneſcat terminus yx^3 in æquatione *reſultante*, quoniam x^3 deſideratur in *data* æquatione. Porro quantitates y, z, & v, ſigillatim inveniendæ ſunt, nempe z & u in terminis ipſius y & quantitatum cognitarum, & y (ſi modo id fieri poſſit) per æquationem cubicam prædictam.

Jam multipl. $xx+yx+z=0$
per $xx-yx+v=0$
prod. $x^4+z \quad -zy \\ -yyxx+vy\ x+vz=0. \\ +v.$

Comparetur hæc cum æquatione datâ $x^4-pxx-qx+r=0$, id eſt æquentur ii termini in quibus eædem poteſtates quantitatis x inveniuntur. Unde 1^mò, $z-yy+v=-p$, $\therefore z+v=yy-p$; 2^dò $vy-zy$

$-zy=-q$, $\therefore v-z=\frac{-q}{y}$; 3tiò erit $vz=r$.

Ex hiſce ut inveniatur z, eſt $\begin{cases} v=yy-p-z \\ v=z-\frac{q}{y} \end{cases}$

$\therefore yy-p-z=z-\frac{q}{y}$, & reducendo $z=\frac{yy-py+q}{2y}$

unde quoniam $v=yy-p-z$; erit $v=yy-p-\frac{y^3+py-q}{2y}=\frac{y^3-py-q}{2y}$. Adeo ut z & v, ſint $\frac{1}{2}yy-\frac{1}{2}p+\frac{q}{2y}$ & $\frac{1}{2}yy-\frac{1}{2}p-\frac{q}{2y}$, & per conſequens æquationes Quadraticæ quas principio finxeramus, ad eas ipſas quas ipſe deſcripſerat *Carteſius* redibunt.

Sed hæc nil proſunt, niſi ſimul æquatio detegatur ad inveniendum valorem quantitatis y. Ut ergo talis obtineatur, multiplicentur in ſe inventi valores quantitatum v & z, & productum ponatur æquale ipſi r (quoniam ex prius dictis eſt $vz=r$) id eſt

$$\frac{y^3-py+q}{2y}\times\frac{y^3-py-q}{2y}=\frac{y^6-2py^4+ppyy-q^2}{4yy}=r,$$

& equatione ordinatâ orietur

$y^6-2py^4+\frac{pp}{4r}yy-qq=0$, quæ ipſa eſt cubica quam ſcribi etiam mandaverat *Carteſius*.

Hanc vero *inveſtigationem* addito exemplo, in *Numeris* illuſtrabimus, ut methodum praxeos tam utilis ac jucundæ clarius percipiant Tyrones.

Sit $x^4-17xx-20x-6=0$, æquatio propoſita.

Fictæ duæ æquationes Quadraticæ $\begin{cases} xx+yx-z=0 \\ xx-yx+v=0 \end{cases}$

Æqua-

Æquatio ex harum multiplicatione resultans

$$\left.\begin{array}{l} x^4 \\ +vxx \\ -yy \end{array}\right\} \begin{array}{l} -z \\ +vy \end{array} \left.\begin{array}{l} +zy \\ \end{array}\right\} x - vz = 0$$

Comparatio inter Terminos duarum Biquadrat. æquat.

$$\left.\begin{array}{l} v-z-yy=-p=-17, \therefore v-z \\ =yy-17. \\ yv+zy=-q=-20, \therefore v+z \\ =\frac{-20}{y}. \\ -vz=-r=-6. \end{array}\right.$$

Duo valores quantitatis v sibi mutuo æquati

$$\left.\begin{array}{l} v=yy-17+z \\ v=\frac{-20}{y}-z \\ \therefore yy-17+z=\frac{-20}{y}-z \end{array}\right.$$

Inventio quantitatis z ex duobus valoribus ipsius v

$$\left.\begin{array}{l} z=\frac{17y-y^3-20}{2y} \end{array}\right.$$

Quantitas v reperta ex z per utramvis æquationem

$$\left.\begin{array}{l} v=yy-17+z= \\ yy-17+\frac{17y-y^3-20}{2y}= \\ \frac{y^3-17y-20}{2y} \\ \text{vel } v=\frac{-20}{y}-z= \\ \frac{-20}{y}-\frac{17y+y^3+20}{2y}= \\ \frac{y^3-17y-20}{2y}, \text{ ut prius.} \end{array}\right.$$

Pro-

Productum $-vz$ ipsi -6 vel $-r$ æquatum, & inde orta æquatio $\left\{\begin{array}{l} \frac{y^3-17y-20}{2y} \times \frac{17y-y^3-20}{2y} \\ =6, \text{ reducendo \& signa mutando, } y^6-34y^4 \\ +313y^2-400=0 \end{array}\right.$

Invento valore yy ex hâc æquatione *Cubicâ*, *viz.* $yy=16$, vel $y=4$, duæ Quadraticæ primò positæ sic determinantur. Erit z $\left(=\frac{17y-y^3-20}{2y}\right.$ $\left.=\frac{17}{2}-8-\frac{12}{8}=\frac{1}{2}-\frac{5}{2}\right)$ $=-2$, ac propterea $xx+yx-z=0=xx+4x+2$. Similiter v $\left(=\frac{y^3-17y-20}{2y}=\frac{8-17-5}{2}=\frac{-14}{2}\right.$ $\left.=-3\right)$, ideoque $xx+yx+v=0=xx+4x+3$.

Jam æquationis $x^2+4x+2=0$, radices sunt $\sqrt{2}-2$, & $-2-\sqrt{2}$. Hæ enim copulatæ cum x & in se mutuo ductæ generabunt ipsam æquationem $x^2+4x+2=0$. Item æquationis $xx+4x+3=0$, radices sunt $\sqrt{7}+2$ & $2-\sqrt{7}$.

Hæ ergo erunt quatuor radices æquationis datæ $x^4+17x^2+20x+6=0$, cum ipsa producatur per multiplicationem x^2+4x+2 per $xx+4x+3$.

Et ad hujus Exempli normam in aliis qui occurrunt Casibus procedendum est.

Schol. (ad *duo Problemata præcedentia*). Quando Radix æquationis Cubicæ *transformatæ*, *possibilis* est, duæ ex radicibus æquationis primò *datæ* sunt Impossibiles. Sed *illa Impossibili existente*, *omnes tres hujus æquationis radices sunt Possibiles*. Porro si datæ æquationis Biquadraticæ, quatuor radices

radices *possibiles* sint, tum æquationis Cubicæ (methodo ostensâ generatæ) tres radices etiam possibiles erunt, adeoque ea ipsa in æquationem cubicam simplicem nunquam transmutari aut ejus radix tali modo obtineri poterit.

Prob. 10. *Limites, intra quos consistunt æquationum Algebraicarum radices, assignare.*

Inutile non erit nec a proposito alienum, priusquam ipsum Problema aggrediamur, hanc observatiunculam præmittere.

Si alicujus æquationis Termini respective multiplicentur per Indices Potestatum incognitæ radicis, hocque continuo fiat & non minus in ipsa æquatione quam in omnibus hisce productis, radicis loco subrogetur quantitas, quæ Radicum Affirmativarum maxima sit major: Dico quod ultimo manebit ubique Quantitas Affirmativa.

Sit æquatio *Quadratica*, signis quomodocunque affecta,

$$\begin{matrix} xx \overset{-a}{\underset{+b}{}} x \overset{-ab}{} = 0. \end{matrix}$$

Multiplicentur singuli termini per exponentes potestatum radicis x, hoc modo

$$\begin{matrix} x^2 \overset{-ax}{\underset{+b}{}} \overset{-ab}{} \\ 2 \quad 1 \quad 0 \end{matrix}$$

T erit

erit productum

$$\begin{matrix} 2x^2 - a \\ + b \end{matrix} x, \text{ vel } 2x - a + b.$$

Pone jam quantitatem puta $a+1$ quæ ipsam radicem affirmativam a excedat, loco x per totam æquationem, & per hoc ipsum productum. Tum que erit $aa+2a+-aa-a+ba+b-ba=$ quantitati *affirmativæ*, simulque $2a+2-a+b$ quantitati affirmativæ æqualis.

Sit etiam æquatio Cubica.

$$\begin{matrix} x^3 - a & + ab \\ - b\ x^2 & - ac\ x + abc = 0. \\ + c & - bc \end{matrix}$$

Si multiplicentur termini per indices potestatum x, hoc est per Progressionem 3. 2. 1. 0, orietur *primo*

$$\begin{matrix} 3x^2 - 2a & + ab \\ - 2b\ x & - ac\ ; \\ + 2c & - bc \end{matrix}$$

& hujus etiam terminis respectivè ductis in terminos progressionis 2. 1. 0. orietur *secundo* $6x-2a-2b+2c$. Supponatur a *omnium radicum affirmativarum maxima*; & scripto $a+1$ pro x per totam æquationem & per duo inde orta producta, erit,

1°. $a^3+3a^2+3a+1-a^2-2a^2-a-ba^2-2ba-b+ca^2+2ca+c+ba^2+ba-ca^2-ca-bca-bc+abc$ = *Quantitati affirmativæ*, hoc eſt, rejectis membris quæ ſe mutuo deſtruunt, erit $a^3+2a^2+1+c-b-ba+ac-bc$ = Quantitati *affirmativæ*, quod patet, cum ex hypotheſi $a > b$, $\therefore ac > bc$, $a^2 > ba$, ergo tota expreſſio affirmativa eſt.

2°. Similiter $3a^2+6a+3-2a^2-2a-2ba-2b+2ca+2c+ab-ac-bc$, ſive $a^2+4a+3-ab-2b+2c+ac-bc$ = quantitati *affirmativæ*.

3°. $6a+6-2a-2b-2c$, hoc eſt, $4a+2c+6-2b$ = quantitati affirmativæ.

Adeo ut Perſpicuum ſit, *quantitatem Affirmativam* orituram ubique, tum in *ipſa æquatione*, quam in *Productis* modo præſcripto generatis.

Unde è contrario, quantitas ea quæ in locum incognitæ radicis ſubſtituta, facit omnes haſce ſummas eſſe Affirmativas; *maximam radicem Affirmativam neceſſario ſuperabit.*

Et *minima* omnium earum quantitatum quæ hoc efficere valent, unus eſt ex *Limitibus* æquationis quæſitis; nempe eſt *limes* ex parte radicum *Affirmativarum.*

Et quæ hoc facit, quando ſigna in locis *alternis* mutantur per omnes haſce ſummas, eſt alter Limes, ſive qui ſtat ex parte radicum *negativarum.*

Hiſce poſitis adducatur in exemplum æquatio

$$x^4 - 17x^2 + 54x - 350 = *$$

Mult.	3	2	1	0
Prod.	$3x^3$	$-34x^2$	$+54x$	

vel dividendo per x, relinquitur.

$$3xx - 34x + 54 *$$

Mult.	2	1	0
Prod.	$6xx$	$-34x$	

vel etiam dividendo per x

$$6x - 34 *$$

Jam inquirendus eſt numerus, qui in omnibus ſummis *(ſtellula notatis)* ſcriptus loco x, relinquat eas *Affirmativas*, hoc eſt efficiat, ut quod reſultat ſit Affirmativum. Et quidem numeros tentando a 1 uſque ad 14, comperitur rem non ſuccedere.

Immò nec ſuccedet, ſi 14 ponatur pro x.
Erit enim $6x - 34 = 84 - 34 = +50$
$3x^2 - 34x + 54 = 588 - 476 + 54 = +116$
$x^4 - 17x^2 + 54x - 350 = 2744 - 3332 + 756 - 350$
$= -182$; unde quoniam hæc ultima provenit negativa, idcirco *limes* major eſt quam 14.

Scribatur ergo 15 pro x, & tum erit
$6x - 34 = 90 - 34 = +56$, &
$3xx - 34x + 54 = 675 - 510 + 54 = +219$

$x^3 - 17x^2 + 54x - 350 =$
$3375 - 3825 + 810 - 350 = + 10.$

Quapropter cum posito $x = 15$, orietur in omnibus summis prædictis numerus affirmativus, erit ipse numerus 15 *limes quæsitus*, hoc est *maxima* radicum affirmativarum *major*.

Notandum vero quod etiam si appareant tres radices affirmativæ in hâc æquatione, duæ tamen ex ipsis sunt *Impossibiles* adeoque æquatio non nisi unâ radice, eaque affirmativa explicabilis est.

CAP. V.

De Problematum Natura in Genere, eorumque variis Diſtinctionibus.

PRimo loco conſideranda eſt (utpote maximè univerſalis) Problematum diſtinctio, in *Determinata* & *Indeterminata*.

Prioris Generis ſunt, quæ *unam* tantum ſolutionem, vel quendam *determinatum* ſolutionum numerum admittunt.

Problemata *Indeterminata* ſunt ea, quibus *Innumera* vel *Infinita Reſponſiones*, ſalvis eorum omnibus Conditionibus, adaptari queant.

Horum frequentia occurrunt Exempla, tum *Arithmetica* tum *Geometrica*.

Ut ſi quæreretur Numerus *Integer Quadratus*, qui dividi poſſit ſine fractione, per 3, 4 & 5.

Vel ſi poſtuletur Triangulum *Rectangulum*, cujus tria Latera ſint *Rationalia*, hoc eſt (comparata ad invicem) ut *Numerus* ad *Numerum*.

In hujuſmodi quæſtionibus, determinationes quæ deficiunt ſuppeditare ſemper licitum eſt per quantitates pro lubitu ſumptas, eo modo quo operis proceſſui maxime expediat. Sed cavendum ne in aſſumptionibus, extra limites quos requirit vel patitur natura quæſtionis, vagemur: Quæſtio (etiamſi indeterminata) conditiones quaſdam habere poteſt, quæ eam *reſtringant*, & intra quoſdam limites quaſi concludant.

Probe

Probe diſtinguendæ ſunt conditiones quæ *reſtringunt* tantum, ab iis quæ abſolute & plene *determinant*.

Uſui porro erit conſiderare, quod harum quæſtionum ſolutiones interdum adeo perfectæ ſint & generales, ut omnes Caſus Poſſibiles comprehendant. Interdum vero magis anguſtæ ac imperfectæ, eâ de causâ quod licet *innumeras* reſponſiones exhibeant, non tamen *omnes* quæ exhiberi poſſunt, vel forſan quæ propoſito maxime inſerviunt. Diſtinguendum ergo eſt inter ſolutionem *veram* & ſolutionem *perfectam*.

Sed ut ex hiſce Generalibus ad particularia & magis utilia deſcendamus, explicanda eſt Problematis Indeterminati, ſenſus famoſior atque nobilior apud *Geometras*.

In *Geometria*, Problema Indeterminatum eſt, quando punctum aliquod quæſitum, quo omnibus Problematis conditionibus plene ſatisfit, indifferenter ſumi poteſt per certam quandam extenſionem, quarumcunque dimenſionum ea ſit, ſive *Linea*, *Superficies*, vel *Solidum*.

Et *hæc ipſa extenſio*, cujus ſingula puncta ad libitum accepta, Problematis conditionibus ſic ſatis dabunt, vulgo nominatur *Locus*, & hujuſmodi Problemata *Localia*, hoc eſt, ad *Locum* ſpectantia.

Locus oritur ex inopiâ materiæ ſive conditionum in Problemate, quâ fit ut tot æquationes non ſuppeditentur, quot requiruntur quantitates incognitæ. Ut ſi *ex. gr.* duæ ſupponerentur quantitates incognitæ in aliquo Problemate, & *adimpletis omnibus ejus conditionibus*, ex iis educi poſſet materia *pro una tantum invenienda*; tunc una incognita neceſſario exprimeretur in terminis *alte-*

rius, & *datarum* quantitatum. Adeoque eam cui (ut loquuntur) non respondet aliqua æquatio, sumire licet ad arbitrium, & inde tot diversos valores invenire pro *altera* incognita quot *huic* ad libitum sumptæ placuerit assingere. Et cum hi valores Infiniti esse possunt, Infinita etiam erunt puncta, per quæ Problematis conditiones accurate impleri poterunt; hoc est, pro talis Problematis solutione, proveniet extensio a *Geometris Locus* communiter appellata.

Locus in Genere *Triplex* est, nempe Locus ad *Lineam*, ad *Superficiem*, ad *Solidum*.

Locus ad Lineam est, quando Punctum Problemati satisfaciens, reperitur in quadam *linea*, sive *recta* sive *curva*; idque propter defectum *unius tantum conditionis*, quæ desideratur ut Problema *penitus determinatum* evadat.

Rursus *Locus ad Lineam*, potest esse *Quadruplex*, viz. *Simplex*, *Planus*, *Solidus*, *Sur-Solidus*, hoc est, *Plusquam-Solidus*.

Simplex, est quando punctum Problemati satisfaciens, reperitur in *Linea Recta*.

Planus, cum idem invenitur in *Circuli circumferentia*.

Solidus, quando in ulla alia Conisectione præter Circulum, nempe vel *Ellipsi*, *Hyperbola*, *Parabola*.

Sur-Solidus, quando in curva *Generis superioris* seu magis *composita* quam sunt Conisectiones.

Quidam

Quidam ſine numerant tantum Locum Planum, Solidum & Sur-ſolidum, ſub Loco Plano interim comprehendentes tam *Lineam rectam* quam *Circuli circumferentiam*. Sed hæc diverſitas ad verba potius ſpectat quam ad res, & proculdubio ſine ullo errore vel periculo, Locus *Planus* vel ad lineam *rectam* extendi, vel ad ſolum *Circulum* reſtringi poterit.

Porro, quod attinet ad *æquationes*, quibus *Loca hæc* exprimuntur, ubi Locus eſt *Simplex* nullibi reperietur in æquatione, Quantitatum incognitarum vel aliquod *productum*, vel *Poteſtas* aliqua, ultra *Primam* ſeu *Simplicem*.

Pro Loco *Plano & Solido*, ſemper habebitur in æquatione vel Quadratum *unius* ex incognitis (ut in *Parabola*) vel *ambarum Quadrata* (ut in *Ellipſi, Circulo* & *Hyperbola*) vel forſan tantum *rectangulum* ſub iis (uti fit in æquationibus ad *Hyperbolam exteriorem* pertinentibus) ſed nullo in Caſu (hoc eſt, *poſt reductionem ad ſimpliciſſimos terminos*, quod ſemper intelligi debet) reperietur ultra *Quadratum* vel (quod *Quadrato* æquipollet) *Planum* ſub duabus Quantitatibus incognitis.

Quando Locus eſt *Sur-ſolidus*, Quantitatum incognitarum *Producta* quædam vel Poteſtates, uſque ad 3 vel 4, ad 5 vel 6, ad 7 vel 8, *&c. dimenſiones* aſſurgent, prout *Linea* quæ Problemati reſpondet, eſt vel *Secundi*, *Tertii*, *Quarti*, *&c. Generis*.

Notandum enim eſt quod Lineæ curvæ in certa *Genera* diſtingui ſolent, per æquationes quas exprimunt relationes, quas *ſingula ſua puncta* habent ad *puncta linea cujuſdam rectæ*. Sic ut quando hæc æquatio non ultra duas dimenſiones aſcendat, curva, cujus natura per eam exprimitur,

Primi

Primi Generis esse dicitur; Cum ad 3 vel 4, *Secundi*, cum ad 5 vel 6, *Tertii*, & sic porro ad Infinitum. Hactenus de *Loco ad Lineam*.

Locus *ad superficiem*, est quando *duabus conditionibus* deficientibus ad puncti alicujus Problematis satisfacientis determinationem, punctum illud sumi poterit per cujusdam *Superficiei* extensionem, nempe sive *Plane* sive aliquo modo Curvæ.

At *Tribus conditionibus* deficientibus ad puncti quæsiti determinationem, *Locus* in quo illud reperitur est *Solidum* quoddam; quodque vel sub *Planis*, vel *curvis*, vel *mixtis* superficiebus, iisque vel *terminatis* vel *indefinite* extensis, concludi poterit.

Et hinc apparet æquationes (ad quas perducuntur Problemata ubi *Locus* est vel *ad superficiem* vel *ad solidum*) *Identicas* esse debere; hoc est tales, ut conditionibus exsecutis & facta reductione debita, una *eademq; quantitas ex utraque æquationis parte inveniatur*; adeoque hujusmodi quæstiones reipsa *Propositiones* seu *Theoremata* esse.

Cum enim, *Loco Linea existente*, una tantum conditio deficiat, ex natura æquationis resultantis indeterminatarum altera alterius expressionem ingredietur: id est per Infinitos valores unius ad libitum sumptæ, totidem valores alteri competentes, determinari queant. At ubi *duæ deficiunt conditiones* (ut in Loco ad *superficiem*) incognitarum una in alterius terminis nunquam exprimenda est: aliter enim *una tantum* desideraretur conditio ut quæstio sit *determinata*, & sic Locus non esset ad *superficiem* sed ad *Lineam*.

Nec

Nec ſimiliter, *tribus conditionibus* deficientibus, obtinenda erit expreſſio unius incognitæ in terminis alterius; ſi enim hoc fieri poſſet, duæ ſolummodo conditiones deficerent, ac tum Locus non ad *ſolidum* ſed ad *ſuperficiem* oriretur.

Neceſſe eſt ergo, terminos in hiſce æquationibus ita ſeſe diſponere, ut *eadem quantitas* (quæcunque illa ſit) *ex utraque parte æquationis ultimo reperiatur*; & præterea nihil.

Omnis enim æquatio talis erit, ut per eam quantitas aliqua incognita vel *poterit* vel *non poterit* exprimi; ac proinde horum una eveniet in æquationibus pro Loco ad ſuperficiem & ad ſolidum.

Sed horum locorum naturæ repugnat, ut quantitatum indeterminatarum una per alteram exprimeretur; quoniam pauciores conditiones eo in caſu deficerent, quam eorum ratio deſiderari patitur. Et propterea nullo modo exprimendæ ſunt per æquationes quæ reſultant. Quicquid enim exprimitur, per quantitates vel *Datas* vel *Indeterminatas* exprimi neceſſe eſt: expreſſiones vero *Datæ* ſive *Determinatæ* ab hoc negotio prorſus abhorrent. Oportet ergo ut quantitates ita ſe tollant utrinque, ut nihil ultimo ſuperſit niſi *Idem ſibi ipſi æquale*.

Hæc vero omnia congruis exemplis Illuſtrabimus, cum de *Locorum Inventione ac Conſtructione*, eorumque *uſibus quibuſdam haud ita vulgatis*, alibi fuſius diſſeremus.

Miſſis ergo Problematis *Indeterminatis*, Quot ſint Problematum *Determinatorum* Genera, Jam videndum eſt.

Hæc

Hæc autem in quatuor Classes prioribus consimiles, vulgo dispescunt *Geometræ*; nimirum in *Simplicia*, *Plana*, *Solida*, *Surfolida* vel *Linearia* (uta quibusdam appellantur.)

Simplex est id Problema, in quo postquam æquatio ultimas subierit reductiones, quantitas incognita *ultra unam dimensionem non ascendit*; adeoque ad ejus *Constructionem* necessariæ sunt tantum *rectæ lineæ*.

Problema *Planum* est, quando (æquatione ad formam simplicissimam perductâ) quantitas incognita ad *Quadratum* duntaxat vel duas dimensiones ascendit; & *constructio* (id est *Solutio Geometrica*) perfici poterit adhibendo *rectas lineas* & *circulos*.

Solidum vero, cum peractâ reductione, & tentatâ quousq; tentari poterit, quantitas incognita ad *tres*, vel ad *quatuor* dimensiones ascendit, ipsaque adeò inveniri, sive *æquatio construi* haud queat, nisi adhibendo *circulum* & aliquam *sectionem conicam*.

Lineare vel *Surfolidum*, cum quantitas incognita invenitur esse dimensionem 5 vel 6, 7 vel 8, 9 vel 10, *&c.* in æquatione *ulterius irreducibili*; cujus ergo constructio, (præter circulum) curvam Generis *Secundi*, *Tertii*, *Quarti*, &c. necessario postulat.

Hinc Problematis *simplicis*, *unica* datur *solutio*, *Plani*, non plures quam *duæ*, nec *Solidi*, quam *quatuor*, nec *Surfolidi* ope circuli & curvæ generis secundi, plures quam *sex*, &c.

Duæ

Duæ enim *rectæ lineæ* in *uno* ſolum puncto ſe invicem ſecant. *Recta* & *circulus*, in *duobus* tantum. *Circulus* & *ſectio conica*, non in pluribus quam *quatuor*. Nec *Circulus* & *Curva Generis ſecundi*, in pluribus quam *ſex*, &c.

Et ex hiſce perducimur ad Problematum ſolidorum Conſtructionem, quam ex Geometriâ Carteſianâ huc tranſtulit, *Vir Clariſſimus* & *undequaque Doctiſſimus*

D. *Edwardus Paget.*

Modus

Modus generalis conſtruendi omnia Problemata ſolida reducta ad Æquationem trium quatuorve dimenſionum.

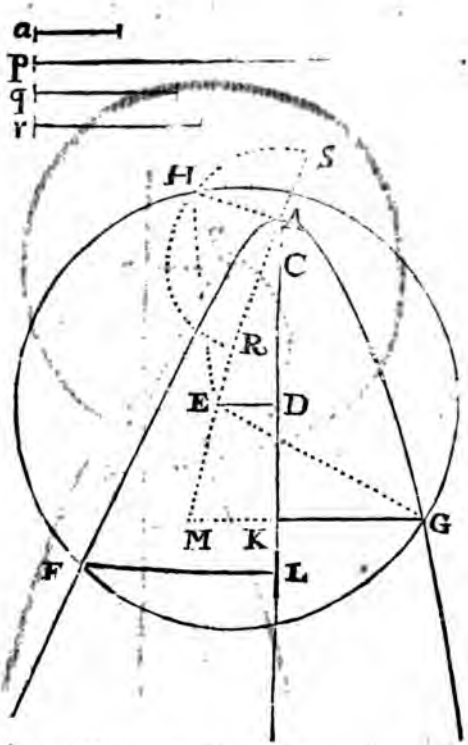
[Ex Geometr. CARTESII, *Lib.* III. *p.* 85.]

POſtquam compertum eſt Problema propoſitum eſſe ſolidum: ſive Æquatio per quam illud quæritur, ad Quadrato-quadratum aſcendat; ſive non altius quam ad Cubum aſſurgat: poteſt ſemper radix ejus inveniri per aliquam trium Conicarum ſectionum, quæcunq; illa tandem ſit; aut etiam per ipſarum particulam aliquam quantumlibet exiguam, nec adhibendo niſi rectas lineas & circulos. Verum ſuffecerit regulam generalem hic adducere inveniendi radices omnes ope Parabolæ, quandoquidem hæc aliquo modo eſt ſimpliciſſima.

Primò igitur tollendus eſt ſecundus Æquationis propoſitæ terminus, niſi jam ante abfuerit, atque ita Æquatio reducenda eſt ad hanc formam $z^3 = * apz. aaq.$ ſi incognita quantitas tres tantum dimenſiones habeat; aut ad hanc $z^4 = * apzz. aaqz. a^3r$, ſi quatuor obtineat dimenſiones; ſeu ſumendo a pro unitate ad has $z^3 = * pz. q.$ $z^4 = * pzz. qz r.$

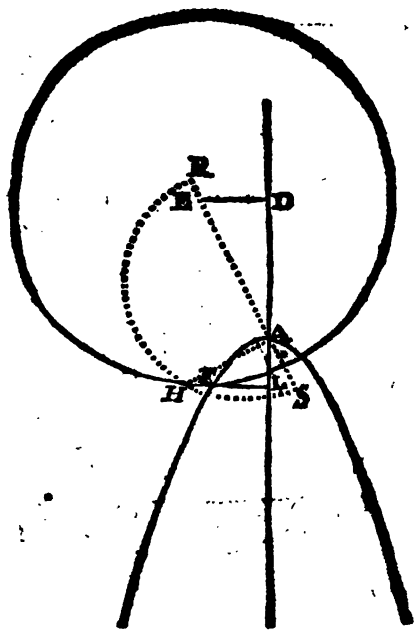
Deinde ſupponendo parabolam FAG jam deſcriptam eſſe, & axem ejus eſſe ACDKL, latuſque rectum a ſeu 1, cujus AC ſit dimidium, & denique

denique punctum C eſſe intra hanc parabolam cujus vertex ſit A. Oportet facere $CD = \frac{1}{2} p$, eamque ſumere in linea AC, continuata verſus C, ſi in Æquatione habeatur $+p$, ſed verſus alteram partem ſi habeatur $-p$. Porrò è puncto D, aut



ex puncto C, ſi non habeatur quantitas p, erigendo ad axem perpendicularem $DE = \frac{1}{2} q$, oportet

tet ex centro E circulum deſcribere FG, cujus ſemidiameter ſit AE, ſi Æquatio tantum Cubica fuerit, hoc eſt, ſi non habeatur quantitas r. Aſt ſi habeatur r, & quidem ſigno $+$ adfecta, oportet ulterius in hac linea AE, producta utrinque, ex una parte ſumere AR$=r$, & ex altera parte AS=lateri recto parabolæ quod eſt $=1$, deſcri-

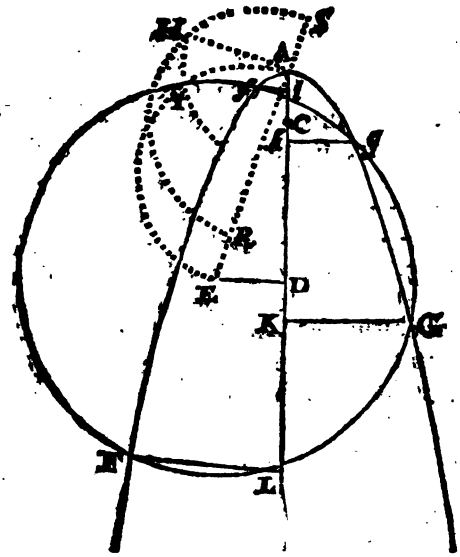


ptoque circulo, cujus diameter RS, erigere AH perpendicularem ad AE, quæ occurrat huic circulo RHS in puncto H; quod illud ipſum eſt per quod alter circulus FHG tranſire debet. Quod

ſi

si vero habeatur $-p$, oportet insuper in alio circulo, cujus diameter est AE, inscribere AI = inventæ AH; inventumq; erit punctum I, per quod primus circulus quæsitus FIG transire debet.

Ubi sciendum, quod circulus hic FG secare vel tangere possit parabolam in 1, 2, 3, aut 4 punctis, a quibus si ad axem demittantur perpendiculares, habebuntur omnes Æquationis radices tam veræ

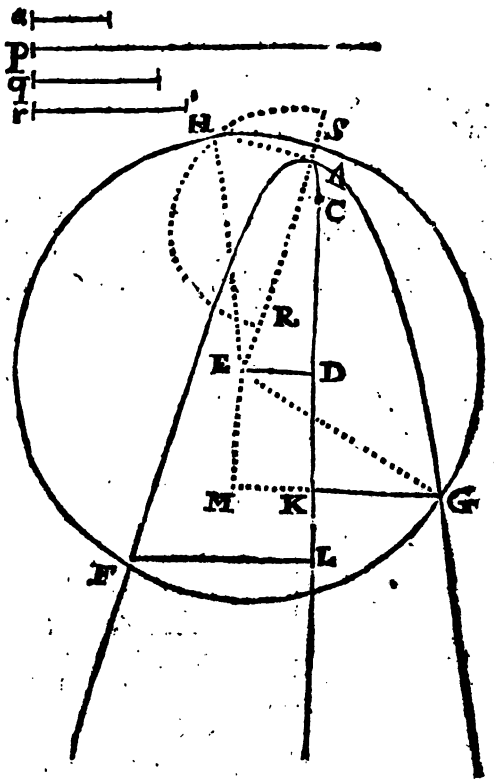


quam falsæ. Nimirum si quantitas q sit adfecta signo $+$, veræ radices erunt illæ harum Perpendicularium, quæ ex eadem Parabolæ parte, qua est E circuli centrum, reperientur, ut FL; & re-

liquæ, ut GK, erunt falſæ. Sed contra ſi hæc quantitas q notata fuerit ſigno —, veræ erunt illæ, quæ ex altera ſunt parte, & falſæ ſeu minores quam nihil, quæ ex parte illa ubi eſt circuli centrum E. Et denique ſi hic circulus non ſecat nec tangit Parabolam in aliquo puncto, indicio eſt Æquationem, nullam admittere radicem ſive veram ſive falſam ſed tantum imaginarias. Adeò ut hæc regula omnium quæ quis exoptare queat, & generaliſſima ſit & perfectiſſima.

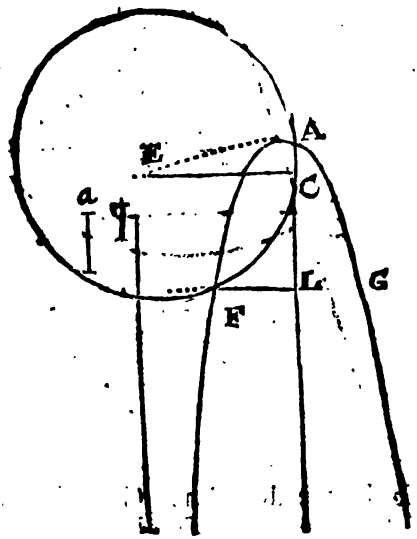
Quorum quidem demonſtratio admodum facilis eſt. Etenim ſi linea GK, per conſtructionem hanc inventa, vocetur z, AK erit zz, propter Parabolam, in qua GK debet eſſe media proportionalis inter AK & latus rectum, quod eſt 1. Deinde, ſi ab AK auferam AC, quæ eſt $\frac{1}{2}$, ut & CD, quæ eſt $\frac{1}{2}p$, relinquetur DK ſeu EM $zz - \frac{1}{2}p - \frac{1}{2}$, cujus quadratum eſt $z^4 - pzz - zz + \frac{1}{4}pp + \frac{1}{2}p + \frac{1}{4}$. Et quia DE ſeu KM eſt $\frac{1}{2}q$, tota GM fit $z + \frac{1}{2}q$, cujus quadratum eſt $zz + qz + \frac{1}{4}qq$, additiſque hiſce duobus quadratis, habebitur $z^4 - pzz + qz + \frac{1}{4}qq + \frac{1}{4}pp + \frac{1}{2}p + \frac{1}{4}$, pro quadrato lineæ GE, quippe quæ baſis eſt trianguli rectanguli EMG.

Sed



Sed quia hæc eadem linea GE eſt ſemidiameter circuli FG, poterit ipſa aliis adhuc terminis explicari. Nimirum ſi ED fuerit $\frac{1}{2}q$, & AD $\frac{1}{2}p$, $+\frac{1}{2}$, EA erit $\sqrt{\frac{1}{4}qq+\frac{1}{4}pp+\frac{1}{2}p+\frac{1}{4}}$, propter angulum rectum ADE. Deinde cum AH ſit media proportionalis inter AS, quæ eſt 1, & AR,

quæ est r, erit ipsa $\sqrt{r}$. Ac denique, propter angulum rectum EAH, quadratum ex HE seu EG, est $\frac{1}{4}qq + \frac{1}{4}pp + \frac{1}{2}p + \frac{1}{4} + r$: adeò ut habeatur Æquatio inter hanc summam & præcedentem. Eadem quippe quæ $zz = \frac{1}{2}p + \frac{1}{2}q + \sqrt{\frac{1}{4}qq + \frac{1}{4}pp + \frac{1}{2}p + \frac{1}{4} + r}$. Unde consequenter liquet, inventam lineam GK, quæ nominata fuit z, Æquationis hujus esse radicem. Quod erat demonstrandum. Et si calculum hunc ad omnes alios hujus regulæ casus applicueritis, mutando signa + & −, prout opus exiget, eodem modo ad quæsitum pervenietis; ita ut illis diutiùs immorari non sit opus.



Si itaque juxta hanc regulam inter lineas a & g duas libeat medias proportionales invenire, nemo

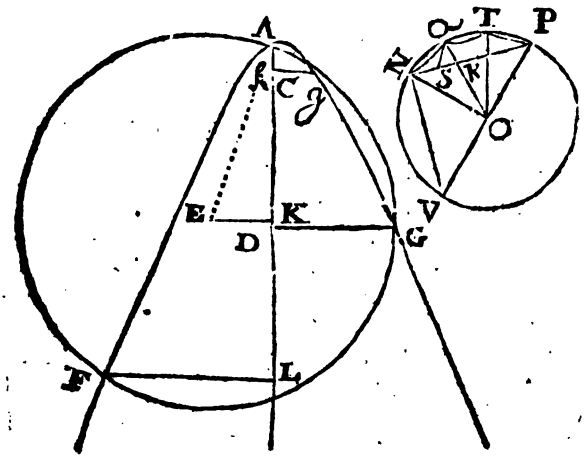
mo ignorat, ponendo z pro una, esse ut a ad z, sic z ad $\frac{zz}{a}$, & $\frac{zz}{a}$ ad $\frac{z^3}{aa}$; ita ut habeatur Æquatio inter q & $\frac{z^3}{aa}$, utpote, $z^3 = **aaq$. Deinde descriptâ Parabolâ FAG, unà cum segmento sui axis AC, quod est $\frac{1}{2}a$, semissis nempe lateris recti, erigenda est ex puncto C perpendicularis CE, æqualis $\frac{1}{2}q$, atque ex centro E per A describendus circulus AF, ut obtineantur FL & LA, duæ mediæ quæsitæ.

Similiter si dividere velimus angulum NOP, sive arcum, portionemve circuli NQTP in tres æquales partes; si sumatur NO$=r$ pro radio circuli, & NP$=q$ pro subtensa arcus dati, ac NQ$=z$ pro subtensa trientis hujus arcus, exsurget Æquatio $z^3 = *3z - q$. Etenim ductis lineis NQ, OQ, & OT; si QS parallela fiat ipsi TO, patet, quod, sicut NO est ad NQ, sic NQ sit ad QR, & QR ad RS; adeo ut, cum NO sit 1, & NQz, QR futura sit zz, & RSz^3. Et quia tantum RS seu z^3 impedit, quo minus linea NP, quæ est q, tripla sit lineæ NQ, quæ est z, habebitur

$$q = 3z - z^3, \text{ vel } z^3 = *3z - q.$$

Deinde descriptâ Parabolâ FAG, in qua CA sit æqualis semissi lateris recti principalis, si sumatur CD$=\frac{1}{2}$, & perpendicularis DE$=\frac{1}{2}q$: secabit circulus FAgG, centro E per A descriptus, hanc Parabolam in tribus punctis F, g, & G, non numerato puncto A, quod est ejus vertex. Id quod indicat in hac Æquatione tres haberi

beri radices, nimirum duas GK & *gk*, quæ veræ ſunt, & tertiam, nempe FL, quæ eſt falſa; atque ex hiſce duabus veris minorem *gk* illam eſſe, quam pro quæſita linea NQ ſumere oportet. Altera enim GK, æqualis eſt ipſi NV, ſubtenſæ



trientis arcus NVP, qui cum reliquo arcu NQP totum circulum complet. Falſa autem FL æqualis eſt duobus hiſce QN & NV ſimul ſumptis, quemadmodum ex calculo facile eſt videre.

Superfluum foret ſi inſiſterem hic aliis exemplis in medium afferendis, cum Problemata omnia, quæ non niſi Solida ſunt, eo reduci poſſint, ut hâc regulâ ad conſtructionem ipſorum non aliter indigeamus, quam quatenus inſervit ad inveniendas duas medias proportionales, aut ad dividendum angulum in tres æquales partes. Quod cognoſcetis, conſiderando, ipſorum difficultater

ſemper

ſemper Æquationibus, quæ ultra Quadrato quadratum non adſcendunt, comprehendi poſſe; & omnes illas, quæ ad Quadrato-quadratum adſcendunt, reduci poſſe ad Quadratum, ope quarundam aliarum, quæ tantum ad Cubum adſcendunt; & tandem, harum ſecundum terminum tolli poſſe. Ita ut nulla earum ſit, quam ad aliquam ex hiſce tribus formis reducere non liceat.

$$z^3 = * - pz + q.$$
$$z^3 = * + pz + p.$$
$$z^3 = * + pz - q.$$

Si autem habeatur $z^3 = * - pz + q$, regula, cujus inventionem Cardanus cuidam, Scipioni Ferreo, tribuit, nos docet, radicem eſſe

$$z = \sqrt{C. + \tfrac{1}{2}q + \sqrt{\tfrac{1}{4}qq + \tfrac{1}{27}p^3}} - \sqrt{C. - \tfrac{1}{2}q + \sqrt{\tfrac{1}{4}qq + \tfrac{1}{27}p}}$$

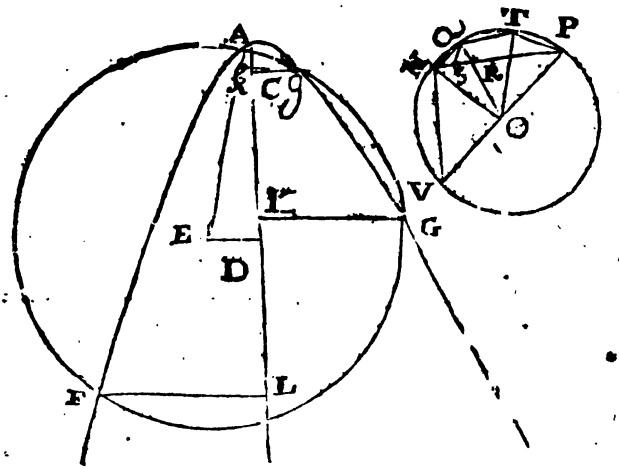
Quemadmodum etiam, ſi habeatur $z^3 = * + pz + q$, & Quadratum ſemiſſis ultimi termini majus ſit Cubo trientis, quantitatis cognitæ penultimi; ſimilis ferme regula nos docet, radicem eſſe

$$z = \sqrt{C. + \tfrac{1}{2}q + \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p^3}} + \sqrt{C. + \tfrac{1}{2}q - \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p}}$$

Unde apparet, quod Problemata omnia, quorum difficultates ad Æquationem unius ex hiſce duabus formis reducuntur, conſtrui ſemper poſſint, ut Conicas ſectiones adhibere non ſit opus, niſi ad extrahendas radices Cubicas ex quibuſdam quantitatibus datis, hoc eſt, ad inveniendas duas medias proportionales inter haſce quantitates & unitatem.

Deinde ſi habeatur $z^3 = * + pz + q$, & Quadratum ſemiſſis ultimi termini non ſit majus Cubo trientis, quantitatis cognitæ penultimi termini;

ſupponendo circulum NQPV, cujus ſemidiameter NO ſit $\sqrt{\frac{1}{3}p}$, hoc eſt, media proportionalis inter trientem quantitatis datæ p & unitatem; tum etiam ſupponendo lineam NP huic Circulo eſſe inſcriptam, quæ ſit $\frac{3q}{p}$, hoc eſt, quæ ſit ad alteram quantitatem datam q, ut eſt unitas ad trientem ipſius p; dividendus tantum eſt uterque arcus NQP, NVP in tres æquales partes, eritque NQ, ſubtenſa trientis unius arcus, una cum NV, ſubtenſâ trientis alterius, æqualis radici quæſitæ.



Denique ſi habeatur $z^3 = *z + pz - q$, ſupponendo rurſus Circulum NQPV, cujus radius NO ſit $\sqrt{\frac{1}{3}p}$, & in quo ſcripta NP ſit $\frac{3q}{p}$: erit NQ,

ſub-

ſubtendens trientem arcus NQP, una ex radicibus quæſitis: & NV, ſubtendens trientem arcus NVP, radix altera. Saltem ſi Quadratum ſemiſſis ultimi termini non excedat Cubum è triente quantitatis cognitæ penultimi termini. Etenim ſi majus eſſet, non poſſet linea NP huic Circulo inſcribi, quippe quæ diametro ejus major foret. Id quod oſtenderet, duas veras radices hujus Æquationis nonniſi imaginarias eſſe, nec ullam realem extare præter falſam, quæ juxta Cardani regulam foret

$$\sqrt{\text{C.}\tfrac{1}{2}q + \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p^3}} + \sqrt{\text{C.}\tfrac{1}{2}q - \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p^3}}.$$

Cæterùm notandum eſt, modum hunc exprimendi valorem radicum per relationem, quam habent ad latera certorum Cuborum, quorum tantùm contentum cognoſcitur, nequaquam magis intelligibilem, neque ſimpliciorem eſſe, quam ſi exprimantur per relationem quam habent ad ſubtenſas certorum arcuum, ſeu Circuli portionum, quarum triplum eſt datum. Ita ut Cubicarum Æquationum radices illæ omnes, quæ per Cardani regulas exprimi nequeunt, æquè clare aut etiam clariùs per modum hic propoſitum exprimi poſſint.

Si enim, exempli causâ, radicem cognoſcere arbitremur hujus Æquationis $z^3 = * + pz + q$: quia ipſam compoſitam eſſe ſcimus ex duabus lineis; quarum una eſt latus Cubi, cujus contentum eſt ſumma, quæ conflatur ex $\frac{1}{2}q$, & ex latere Quadrati, cujus contentum eſt $\frac{1}{4}qq - \frac{1}{27}p^3$; & altera latus alterius Cubi, cujus contentum eſt differentia quæ eſt inter $\frac{1}{2}q$, & latus Quadrati, cujus contentum eſt $\frac{1}{4}q - \frac{1}{27}p^3$ (quod illud omne eſt, quod ex Cardani regula addiſcimus);

Dubi-

Dubitandum non est, quin æque distincte aut etiam distinctiùs radix hujus $z^3 = * + pz - q$ cognoscatur, si ea consideretur inscripta Circulo, cujus semidiameter sit $\sqrt{\frac{1}{3}p}$, in quo pro subtensa arcus intelligatur, cujus tripli subtensa sit $\frac{3q}{p}$. Quinetiam hi termini prioribus illis multò minus sint intricati, & qui etiam multò breviores reddentur, si peculiari aliquâ notâ ad exprimendas hasce subtensas, quemadmodum fit notâ $\surd$ C. ad exprimendum latus Cubicum, uti velimus.

Et hactenus de methodo *Cartesianâ*, propriis Autoris verbis descriptâ per Clarissimum D. *Paget*

Ex duabus autem considerationibus, *regula hæc pro Construendis Problematis Solidis*, Autori in mentem venire potuisset; unâ *Algebraica*, alterâ ex *Geometria* depromptâ.

Si enim *Circulus Parabolæ* occurrat quatuor in punctis, & ab intersectionibus perpendicula ad *Axem* demittantur; erit summa earum ab unâ *axis* parte, æqualis summæ reliquarum quæ ab altera parte *axis* existunt. Et si forte *Circulus Parabolam contingat*, tum *dupla* ordinata à *puncto contactus*, duabus reliquis *simul sumptis* æqualis erit. Et hanc proprietatem non modo ad Parabolam pertinere, sed etiam ad Ellipses, & Hyperbolas intersectas à circulo, vel aliis sectionibus demonstrat *Celeberrimus Geometra De La Hire*, Lib. 5. *Sect. Conic.*

Hâc autem Parabolæ Proprietate bene perspectâ, sequebatur inde, quod si radices *Negativæ* & *Affir-*

Affirmativæ alicujus æquationis se mutuò destruerent, vel si summa Radicum Negativarum Affirmativarum summæ æqualis esset (id est, si *secundus æquationis terminus tolleretur*) tum istius æquationis radices per hujusmodi *Perpendicula*, ab intersectionibus ad *axem Parabolæ* demissa, accuratè explicari posse: adeo ut ad *Regulam pro constructione* absolvendam, nihil amplius supereſſet, nisi ut detegerentur *Positio* & *Magnitudo* istius circuli qui Parabolam ita secaret, ut ordinatæ ab intersectionibus ductæ, datæ æquationis radices explicarent.

Hic vero circulus *Positione* & *Magnitudine* cognosceretur, si centrum ejus Positione daretur, & præterea Magnitudo Radii; Hoc est, si reperiri posset, quo *Radio*, & ad quam *distantiam* ab axe Parabolæ eum describere oporteret.

Hunc in finem lineæ AD, DE, EH, EI, ut prorsus ignotæ considerandæ essent, & earum valores in terminis cognitis datæ æquationis inveniendæ.

Sit AD=x, DE=y, EH vel EI=v, KG=z (ut antea).

Tum erit AK=$\frac{zz}{a}$, DK=EM=$\frac{zz}{a}$—x, MG =z+y, & per consequens EG^q (=EM^q+MG^q) =$\frac{z^4}{aa}$—$\frac{2xzz}{a}$+xx+yy+2yz+zz (=EH^q) =vv, vel æquationem reducendo & ordinando z^4=

$$+\frac{2ax}{aa}zz - 2aayz + \begin{matrix} +aavv \\ -aaxx \\ -aayy \end{matrix}$$

Comparetur hæc cum datâ quâlibet æquatione (*v. gr.*)

$t^4 = +apt^2 - aaqt + a^3r$, & ex terminorum homologorum collatione sequentes æqualitates orientur, *viz.* 1. $2ax - aa = +ap$, $\therefore x = \frac{1}{2}a + \frac{1}{2}p$,

2. $2aay = aaq$, $\therefore y = \frac{1}{2}q$,

3. $aavv - aaxx - aayy = a^3r$, $\therefore vv = ar + xx + yy$, & substituendo valores repertos ipsarum x & y, erit $vv = ar + \frac{1}{4}aa + \frac{1}{2}ap + \frac{1}{4}pp + \frac{1}{4}qq$.

Hinc jam patet quod pro lineâ AD sumere oportet in axe Parabolæ $\frac{1}{2}a + \frac{1}{2}p$, sive dimidium Lateris Recti una cum dimidio quantitatis cognitæ 3^tii^ termini æquationis, utpote signo positivo hic affecti. Præterea longitudo ipsius DE (quæ centrum determinat) accipienda est æqualis semissi quantitatis cognitæ termini 4^ti^.

Ultimò Radius EH vel *v* talis esse debet, ut ejus Quadratum sit æqualis Quadrato lineæ AE ($= \frac{1}{4}aa + \frac{1}{2}ap + \frac{1}{4}pp + \frac{1}{4}qq$) una cum Quadrato quodam quod æquatur rectangulo *ar*; nimirum in lineâ AE producta sumere oportet ex unâ parte, à puncto A, longitudinem ipsius *r*; & ex alterâ longitudinem *Lateris Recti a* sive 1, & faciendo lineam SR ex utrisque compositam, Diametrum Semicirculi, ex prædicto puncto A, perpendicularem AH erigere usque ad semicirculum, quæ ergo linea AH, media proportionalis erit inter duo diametri segmenta, adeoque quadratum ejus ipsi rectangulo *ar* æquale, unde linea EH dat magnitudinem Radii quæsiti.

Si vero quantitas *r* signo negativo adfecta fuisset, tunc invenissemus $vv = xx + yy - ar$, adeoque quadratum radii quæsiti æquale *differentiæ* Quadratorum à lineis AE & AH.

Ex quo discimus, quod stante priore constructione, & descripto super Diametro AE semicirculo, oportuisset lineam AI ipsi AH æqualem in eo

eo coaptare, & tum circulum centro E, (ut prius) Radio vero EI describere.

Si quantitas r, fingatur nihilò æqualis, vel penitus abesse, tum æquatio convertetur in *Cubicam*, & proinde ejus constructio sic facillimè deducitur.

Quoniam $a^3r=0$, $\therefore vv=xx+yy$, $=AE^q$, & per consequens circulus per verticem A Parabolæ transire debet.

Et hæc sunt quæ in methodo *Cartesiana* ad horum Problematum constructionem requiruntur.

Hinc vero consequitur alia methodus hujusmodi Problemata construendi, in quâ perpendicula ab intersectionibus Circuli & Parabolæ demissa, non ad *ipsum Axem*, sed ad *Diametrum* quandam (quæ axi ex Parabolæ naturâ Parallela est) referuntur.

Sit enim æquatio $x^4+px^3-qx^2-rx+s=0$, quæ ut *more Cartesiano* construatur, tollatur terminus 2*dus* ponendo nimirum $z=x+\frac{1}{4}p$, $\therefore x=z-\frac{1}{4}p$, quo ubique substituto pro x, oritur

$$\begin{array}{rrrrrl}
z^4 & -z^2p & +\frac{3}{8}z^2p^2 & -\frac{1}{16}zp^3 & +\frac{3}{256}p^4 & =x^4 \\
 & +z^3p & -\frac{3}{4}z^2p^2 & +\frac{3}{16}zp^3 & -\frac{1}{64}p^4 & =x^3 \\
 & & -qz^2 & +\frac{1}{2}qpz & +\frac{1}{16}qp^2 & =-qx^2 \\
 & & & -rz & +\frac{1}{4}rp & =-rx \\
 & & & & +s &
\end{array}$$

Id est, ablatis terminis contradictoriis,

$$z^4 \begin{array}{l} -\frac{3}{8}p^2 \\ -q \end{array} zz \begin{array}{l} +\frac{1}{8}p3 \\ +\frac{1}{2}qp \\ -r \end{array} z \begin{array}{l} -\frac{3}{256}p^4 \\ -\frac{1}{16}qp^2 \\ +\frac{1}{4}rp \\ +s \end{array} = 0$$

Id

Ergo *Cartesii* linea *verticalis* AD, constat ex AC sive $\frac{1}{2}$, & quantitate $-\frac{1}{16}p^2-\frac{q}{4}$ *deorsum* ponenda a puncto C: & *Normalis* DE$=\frac{1}{16}p^3+\frac{1}{4}qp-\frac{1}{2}r$ sumenda *dextrorsum* a puncto D, posito $r>\frac{1}{8}p^3+\frac{1}{2}qp$.

Si ducatur linea Parabolam tangens in vertice A, & in ea sumatur distantia $\frac{1}{4}p$ ad dextram axis partem perque hujus finem linea axi parallela protrahatur, erit pars hujus Diametri exterior, viz. inter curvam & tangentem verticalem intercepta $=\frac{1}{16}p^2$ (utpote abscissa ordinatæ $\frac{1}{4}p$ respondens) Quapropter segmentum Diametri, curvam & lineam DE *productam*, interjacens, æqualis erit AD$-\frac{1}{16}p^2=\frac{1}{2}+\frac{1}{16}p^2+\frac{q}{4}-\frac{1}{16}p=\frac{1}{2}+\frac{q}{4}+\frac{1}{8}p^2$: Quæ est *Cl. Bakeri* Regula pro sua linea *verticali* determinanda, in hæc æquationis formulâ.

Denique quoniam DE$=\frac{1}{16}p^3+\frac{1}{4}qp-\frac{r}{4}$, ergo distantia inter punctum E & Diametrum prædictam necessario est æqualis $\frac{1}{16}p^3+\frac{1}{4}qp-\frac{r}{4}+\frac{1}{4}p$; quæ itidem est Authoris modò memorati formula pro inveniendâ suâ linea *transversa* sive ad Diametrum *normali*.

Et ex hisce duabus, conflatur ejus *Regula Centralis*, sive regula reperiendi locum centri, unde circulus descriptus, per verticem *Diametri* in *Cubicis* æquationibus, at per punctum eodem modo quo prius investigatum, in *Biquadraticis* transiens, in tot punctis Parabolam secabit, quot reales æquatio radices habuerit.

Rogandus est Lector, ut pauca hæc quæ sequuntur, ad Loca infra notata in *Appendice* referre dignetur.

Ad Pag. 38. Lin. 13. Post verbum *invenire*, pergat hoc modo.

Vel

Vel ex æquatione quæ resultat, aliter inveniri poterit expressio quantitatis y in terminis ipsius v, sed per seriem (etsi unicam, tamen) multo magis compositam & perplexam.

Ad Pag. 41. Lin. 2. Post hæc verba *unde patet via*, adde *sequentia*, sed duo etiam modi exprimendi quantitatem y per v, & *cognitas* sic aliter obtineri possunt.

Quoniam $\overline{1+y}^{n} = \overline{1+v}^{s}$, $\therefore 1+y = \overline{1+v}^{\frac{s}{n}}$, unde habetur y per seriem potestatum ipsius v.

Iterum, $1+y^{\frac{s}{n}} = \overline{1+v}^{s}$, $\therefore 1+y = \overline{1+v}^{m}$, & inde reperitur altera series pro y. Quibus duabus seriebus inter se æquatis, habetur æquatio pro v.

Sed malo *unam Regulam Generalem in omni Casu* adhibere, ideoque per *Methodum pro extrahenda radice ex æquatione Infinita* semper procedere, etiamsi in casu adducto, possibile sit sine eâ, ac ope solius *Theorematis Binomii* rem perficere.

Aliud vero exemplum lubet adjungere precedentibus, ex quo magis eluceret usus harum Methodorum in separandis & exprimendis quantitabus incognitis.

Sint Æq.
$$\begin{cases} az+bz^{2}+cz^{3}, \&c. = \overline{dy+ey^{2}+fy^{3}}^{n}, \&c. \\ gy+hy^{2}+iy^{3}, \&c. = kx+lx^{2}+mx^{3}, \&c. \\ nx+ox^{2}+px^{3}, \&c. = \overline{qv+rv^{2}+sv^{3}}^{m}, \&c. \\ lv+nv^{2}+qv^{3}, \quad = \overline{cz+dz^{2}+bz^{3}}^{s}, \&c. \end{cases}$$

Nam in 1ma æquatione, ope *Theorematis Infinitinomii* (quod etiam Laudato D. *De Moivre* debemus) elevatur series Infinita $dy+ey^{2}+fy^{3}$, &c. ad Potest. n.

Et

inde per alterum Theorema (pro extrahendi-
lice ex æquatione Infinita) habetur z in serie
tâ Potestatum ipsius y.
rsus eodem planè modo ex 4ta æquatione lv
$+qv^2$, &c. $= cz + dz^2 + fz^3$, &c. exprimitur

o cum habeatur z per duas series infinitas
ssa, nempe Potestatum y & v, habebitur
y in v.
iliter ex 2da æquat. $gy + by^2 + iy^3$, &c. $=$
$x^2 + mx^3$, &c. exprimitur x in y, & per
quat. $nx + ox^2 + px^3$, &c. $= qv + rv^2 + sv^3$, &c.
ur etiam x in v. Quapropter per duas se-
nfinitas quibus exprimitur x tum in y tum in
enietur altera equatio quâ y *secunda vice*
netur in terminis ipsius v. Et ex duabus
seriebus totidem valores quantitatis y exhi-
ous, oritur æquatio determinata qua quan-
plenè exprimitur.

F I N I S.

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